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Generalized Maximum Fuzzy Entropy Methods with Applications on Wind Speed Data

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Abstract: This study is connected with new Generalized Maximum Fuzzy Entropy Methods (GMax(F)EntM) in the form of MinMax(F)EntM and MaxMax(F)EntM belonging to us. These methods are based on primary maximizing Max(F)Ent measure for fixed moment vector function in order to obtain the special functional with maximum values of Max(F)Ent measure and secondary optimization of mentioned functional with respect to moment vector functions. Distributions, in other words sets of successive values of estimated membership function closest to (furthest from) the given membership function in the sense of Max(F)Ent measure, obtained by mentioned methods are defined as $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ which is closest to a given membership function and $(\mathrm{MaxMax(F)Ent})_{\mathrm{m}}$ which is furthest from a given membership function. The aim of this study consists of applying MinMax(F)EntM and MaxMax(F)EntM on given wind speed data. Obtained results are realized by using MATLAB programme. The performances of distributions $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ and $(\mathrm{MaxMax(F)Ent})_{\mathrm{m}}$ generated by using Generalized Maximum Fuzzy Entropy Methods are established by Chi-Square, Root Mean Square Error criterias and Max(F)Ent measure.

Keywords: Maximum fuzzy entropy measure, Generalized maximum fuzzy entropy methods, Moment vector functions, Membership function.

1. Introduction

Entropy is a measurement of the degree of uncertainty. Over the years, the concept of entropy has been applied in several problem areas within the social sciences, especially in statistics, economics and engineering [1]. The use of entropy in statistics is related with the information theory. Shannon [2] has introduced entropy as a measure of uncertainty of random variable X in the following form

$$H(X) = -\sum_{i=1}^{n} p_i log(p_i) \ , i = 1,2,\dots,n \ ,$$

where $p_i = P\{X = x_i\}$, $\sum_{i=1}^{n} p_i = 1$.

$H(X)$ has the following properties: H is non-neagative; $H = 0$ if and only if $p_i = 1\,, i = 1,2,,\dots,n$ and H reaches its maximum value $H_{max} = logn$ when $p_1 = p_2 = \cdots = p_n = 1/n$.

The measure of uncertainty is adopted as a measure of information. Therefore, the measures of fuzziness are known as fuzzy information measures. The measure of a quantity of fuzzy information obtained from a fuzzy set or fuzzy system is known as fuzzy entropy. Here, it should be noted that the meaning of fuzzy entropy is quite different from the classical Shannon entropy because no probabilistic concept is needed in order to define it. This is due to the fact that fuzzy entropy contains vagueness and ambiguity uncertainties, while Shannon entropy contains the randomness uncertainty [3].

The fuzzy entropy is defined using the concept of membership function. Zadeh [4] suggested that fuzzy entropy guantifies measurement of the uncertainty associated with each fuzzy value as a weighted Shannon entropy. Then, fuzzy entropy has been studied by many researchers.

De Luca and Termini [5] first initialized the entropy of a fuzzy set by using Shannon probabilistic entropy measure of fuzzy entropy for a fuzzy set containing finite number elements. Kauffman [6] proposed that

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the entropy of a fuzzy set can be measured through the distance between the fuzzy set and its nearest set. After that, Knopfmacher [7] extended the definition of fuzzy entropy made by De Luca and Termini and Kauffman, respectively. Yager [8] defined another kind of fuzzy enropy measure by using distance of the fuzzy set and its complement. Kosko [9] introduced the fuzzy entropy based on the fuzzy set theory and distances between them. Also, there is a lot of research studies concerning the definition of fuzzy entropy and its applications such as Bhandari and Pal [10], Pal and Pal [11].

Measure of fuzziness indicates the degree of fuzziness of a fuzzy set. The entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. The fuzzy entropy is defined by using the concept of membership function. The fuzzy entropy proposed by De Luca and Termini is shown the following formula,

$$H(A) = -\sum_{i=0}^{n}[\mu_A(x_i) log\mu_A(x_i) + (1 - \mu_A(x_i))\, log\big(1 - \mu_A(x_i)\big)].$$

where A is a fuzzy set, $\mu_A(x)$ is membership function and $\mu_A(x_i)$ are the fuzzy values.

This fuzzy entropy has following properties: $H(A) = 0$ if and only if A is crisp set and $H(A)$ is maximum if and only if $\mu_A(x_i) = 0.5\,, \forall x_i \in A$.

After the development of given in [5], a large number of measures of fuzzy entropy were discussed, characterized and generalized by various authors. Parkash, Sharma and Mahajan [12] introduced new measures of weigted fuzzy entropy including two moment conditions and obtained relationships among these measures with respect to their applications for the study of maximum weighted fuzzy entropy principle. Guo and Xin [13] have extended Zadeh's idea [4] to improve some new generalized entropy formulas for fuzzy sets.

For practical problems without sufficient information, the determination of uncertainty distributions of fuzzy values is an important problem in the fuzzy set theory and needs to be estimated with accessible information about fuzzy values. For a fuzzy set, it can't be obtained the membership function clearly. At this point, Maximum Entropy Method (MaxEnt) proposed by Jaynes can successfully solve this problem by maximizing the Shannon entropy measure subject to moment constraints. Following the Maximum Entropy Method, in fuzzy set theory, it is introduced the fuzzy entropy measure which maximizes the value of fuzzy entropy subject to moment constraints [12].

The aim of this paper consists of realizing an application of Maximum Fuzzy Entropy Method (Max(F)EntM) and Generalized Maximum Fuzzy Entropy Methods (GMax(F)EntM) for Max(F)Ent measure subject to $m + 1$ moment constraints. It should be noted that Max(F)EntM and GMax(F)EntM are developed in our another study. Following the Generalized Maximum Entropy Methods [14-17], in fuzzy theory, we introduce Generalized Maximum Fuzzy Entropy Methods and their solutions in the form of distributions $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ which is closest to a given membership function and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ which is furthest from a given membership function in the sense of Max(F)Ent measure.

This paper organized as follows. In Section 2, it is introduced the Max(F)Ent functional for fuzzy values, the process of determining $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distributions is given. In Section 3, an application on wind speed data is fulfilled by using GMax(F)EntM in detail. Finally, the main results obtained in this study are summarized.

2. Max(F)Ent Functional

The Maximum Fuzzy Entropy Method (Max(F)EntM) is a new approach to obtain a membership function via moment constraints in fuzzy set theory. According to this method, we introduced a special functional on the given compact set of moment vector functions. By virtue of the moment vector functions, which give the least and the greatest values to the mentioned functional, a process of obtaining $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$ distributions generated by GMax(F)EntM is theoretically defined. The $(\mathrm{MinMax(F)Ent})_m$ distribution is the closest to the distribution of the given membership function and the $(\mathrm{MaxMax(F)Ent})_m$ distribution is the furthest from the distribution of given membership function in the sense of Max(F)Ent measure. The problem of maximizing fuzzy entropy measure defined by De Luca and Termini by using Shannon probabilistic entropy measure, we shall use in the following form

$$H(A) = -\sum_{i=0}^{n}[\mu_A(x_i)ln\mu_A(x_i) + (1 - \mu_A(x_i))\,ln\big(1-\mu_A(x_i)\big)] \quad (1)$$

subject to constraints

$$\textstyle\sum_{i=0}^{n}\mu_A(x_i)\,g_j(x_i) = \mu_j\ ,j = 0,1,2,\dots,m \quad (2)$$

where $g_0(x) \equiv 1$; $\mu_j, j = 0,1,2,\dots,m$ are moment values of $\mu_A(x_i)\ ,i = 0,1\ ,\dots,\ n$ with respect to moment functions $g_j(x)\ ,j = 0,1,2,\dots,m\,;m < n$. This problem has a solution

$$\mu_A(x_i) = \frac{1}{1+e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}}\ ,i = 0,1,\dots,n \quad (3)$$

where $\lambda_j\ ,j = 0,1,\dots,m$ are Lagrange multipliers. We note that mentioned problem for entropy optimization measure is suggested and solved in [14-17]. In application, one of the obtaining methods of membership values is used in the form $\mu_i = \frac{Frq_i}{\sum_{i=0}^{n}Frq_i}\ ,i = 0,1,\dots n$ [18].

If Eq. (3) is substituted in Eq.(1), the maximum value of Max(F)Ent measure (1) is obtained:

$$maxH_A = U(g) = -\sum_{i=0}^{n} ln\frac{e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}}{1+e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}} + \sum_{j=0}^{m}\lambda_j\mu_j \ . \quad (4)$$

If distribution $\mu(x) = \big(\mu_A(x_0),\mu_A(x_1),\dots,\mu_A(x_n)\big)$ is calculated from the data, the moment vector value $\mu = (\mu_0\ ,\mu_1,\dots,\mu_m)$can be obtained for each moment vector function $g(x) = \big(g_0(x),g_1(x),\dots,g_m(x)\big)$. Therefore, $maxH_A$ is considered as a functional of $g(x)$ and called Max(F)Ent functional. Thus, we use the notation $U(g)$ to denote the maximum value of $H(A)$ corresponding to $g(x) = \big(g_0(x),g_1(x),\dots,g_m(x)\big)$.

2.1 $(\boldsymbol{MinMax(F)Ent})_{\boldsymbol{m}}$ *and* $(\boldsymbol{MaxMax(F)Ent})_{\boldsymbol{m}}$ *Distributions*

Generalized Maximum Fuzzy Entropy Distribution indicated as $(\mathrm{MinMax(F)Ent})_m$ is closest to a given membership function and distribution indicated as $(\mathrm{MaxMax(F)Ent})_m$ is furthest from a given membership function in the sense of Max(F)Ent measure. Solving the MinMax(F)Ent and MaxMax(F)Ent problems require to find vector functions $\big(g_0,g^{(1)}(x)\big)$, $\big(g_0,g^{(2)}(x)\big)$ where $g_0(x) \equiv 1, g^{(1)} \in K_{0,m}$, $g^{(2)} \in K_{0,m}$ minimizing and maximizing functional $U(g)$ defined by (4), respectively. It should be noted that $U(g)$ reaches its minimum (maximum) value subject to constraints (2) generated by function $g_0(x)$ and all m −dimensional vector functions $g(x), g \in K_{0,m}$. In other words, minimum (maximum) value of $U(g)$ is least (greatest) value of values $U(g)$ corresponding to $(g_0(x),g)$, $g \in K_{0,m}$. In other words, $(\mathrm{MinMax(F)Ent})_m$ $((\mathrm{MaxMax(F)Ent})_m)$ is distribution giving minimum (maximum) value to functional $U(g)$ along of all distributions generated by $\binom{r}{m}$ number of moment vector functions $(g_0(x), g), g \in K_{0,m}$. Therefore, we denote mentioned distributions in the form of $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$.

Let K be the compact set of moment vector functions $(g_0(x),g)$, $g \in K_{0,m}.U(g)$ reaches its least and greatest values in this compact set, because of its continuity property. For this reason,

$$\min_{g\in K} U(g) = U\big(g^{(1)}\big)\,;\ \max_{g\in K}U(g) = U\big(g^{(2)}\big)$$

Consequently,

$$U\big(g^{(1)}\big) \le U\big(g^{(2)}\big).$$

If $\big(g_0\ ,g^{(1)}(x)\big)$ vector function gives the minimum value to $U(g)$, then distribution of

$\mu^{(1)} = \left(\mu^{(1)}(x_0), \mu^{(1)}(x_1), \ldots, \mu^{(1)}(x_n)\right)$ is called $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ distribution corresponding to $\left(g_0, g^{(1)}(x)\right)$.

In a similar way, if $\left(g_0, g^{(2)}(x)\right)$ vector function gives the maximum value to $U(g)$, then distribution of $\mu^{(2)} = \left(\mu^{(2)}(x_0), \mu^{(2)}(x_1), \ldots, \mu^{(2)}(x_n)\right)$ is called $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distribution corresponding to $\left(g_0, g^{(2)}(x)\right)$.

The existence of distributions $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ is proved by the following theorem.

Existence Theorem. Let us the following conditions are satisfied:

(1) Moment functions $g_j(x), j = 0,1,2,\ldots,m$ are linearly independent;

(2) The inequality $n > m$ is satisfied;

(3) Moment values $\tilde{\mu}_j, j = 0,1,\ldots,m$ are obtained by virtue of given fuzzy values $\tilde{\mu}_A(x_i), i = 0,1,\ldots,n$ and $g_j(x), j = 0,1,\ldots,m$ in the form of equalities

$\sum_{i=0}^{n} g_j(x_i)\, \tilde{\mu}_A(x_i) = \tilde{\mu}_j, \; j = 0,1,\ldots,m.$

Then, Maximum Fuzzy Entropy Problem (Max(F)EntP) which consists of maximizing fuzzy entropy measure (1) with respect to membership functions $\mu_A(x)$ with finite number of the fuzzy values $\mu_A(x_i), i = 0,1,\ldots,n$ subject to constraints (2) has a solution $\left(\mu_A(x_0), \mu_A(x_1), \ldots, \mu_A(x_n)\right)$.

3. Application of MinMax(F)Ent and MaxMax(F)Ent Methods on Wind Speed Data

The wind energy is one of the most significant and rapidly developing renewable energy sources in the world and it provides a clean energy resource. For this reason, the probability distribution of wind speed is one of the most important wind characteristics for assessment of wind energy potential and for the performance of wind energy conversion systems [19]. In this section, it is shown that $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distributions obtained by Generalized Maximum Fuzzy Entropy Methods (GMax(F)EntM) is suitable for the assessment of the wind energy potential.

In this study, fuzzy values of wind speed data given in Table 3 is investigated by virtue of Generalized Maximum Fuzzy Entropy Methods (GMax(F)EntM) in the form of MinMax(F)EntM MaxMax(F)EntM. For this reason, it is required to obtain distributions $(\mathrm{MinMax(F)Ent})_\mathrm{m}$, $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ according to given fuzzy data. This problem is solved in the following form. Firstly, Max(F)Ent characterizing moments of given moment functions according to fuzzy data is determined. Then, Max(F)Ent distributions subject to each of Max(F)Ent characterizing moments is calculated. Hereafter, distributions generated by GMax(F)EntM corresponding to selected Max(F)Ent characterizing moments are obtained. Obtained results are realized by using MATLAB programme. The performances of distributions generated by Generalized Maximum Fuzzy Entropy Methods are evaluated by statistical critereas as Root Mean Square Error, Chi-Square and Max(F)Ent measure.

In order to obtain $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ $(m = 1,2)$ distributions, we should choose the moment vector functions giving the maximum and minimum values to the Max(F)Ent functional $U(g)$. Here, we used the moment functions

$$g_0(x) = 1, g_1(x) = \sqrt{x}, g_2(x) = lnx, g_3(x) = ln(1+x), g_4(x) = ln(1+x^2).$$

According to suggested method, $K_0 = \{g_0, g_1, g_2, g_3, g_4\}$ and all combinations of r elements of K_0 taken m elements at a time are denoted by $K_{0,m}$. It should be noted that $m = 1,2$ are the most suitable values for the considered data.

For $m = 1$, $K_{0,1} = \{(1, \sqrt{x}), (1, \ln x), (1, \ln(1+x)), (1, \ln(1+x^2)\}$.

From Table 2, it is shown that $(g_0, g^{(1)}) = (1, ln(1+x))$, $g^{(1)} \in K_{0,1}$ gives to least value to $U(g)$, consequently corresponding distribution is $(\text{MinMax(F)Ent})_1$ and $(g_0, g^{(2)}) = (1, \sqrt{x})$, $g^{(2)} \in K_{0,1}$ gives to greatest value to $U(g)$, consequently corresponding distribution is $(\text{MaxMax(F)Ent})_1$. In a similar way,

For $m = 2$, $K_{0,2} = \{(1, \sqrt{x}, \ln x), (1, \sqrt{x}, \ln(1+x)), (1, \sqrt{x}, \ln(1+x^2)), (1, \sqrt{x}, \ln(1+x)), (1, \ln x, \ln(1+x^2)\}$.

From Table 3, it is shown that $(g_0, g^{(1)}) = (1, \sqrt{x}, lnx)$, $g^{(1)} \in K_{0,2}$ gives to least value to $U(g)$, consequently corresponding distribution is $(\text{MinMax(F)Ent})_2$ and $(g_0, g^{(2)}) = (1, \sqrt{x}, ln(1+x$, $g^{(2)} \in K_{0,2}$ gives to greatest value to $U(g)$, consequently corresponding distribution is $(\text{MaxMax(F)Ent})_2$.

Now, in order to obtain the performance of the $(\text{MinMax(F)Ent})_\text{m}$ and $(\text{MaxMax(F)Ent})_\text{m}$, $m = 1,2$ distributions, we use various criterias as Root Mean Square Error (RMSE), Chi-Square (χ^2) and Max(F)Ent measure (H). The acquired results are demonstrated in Table 4-5.

Tables 4-5 show that in the sense of RMSE and χ^2 criterias each of $(\text{MaxMax(F)Ent})_m$, (m=1,2), distributions is better than each of $(\text{MinMax(F)Ent})_\text{m}$ (m=1, 2) distributions. Furthermore, $(\text{MaxMax(F)Ent})_1$ distribution provides better performance along $(\text{MaxMax(F)Ent})_m$, m = 1, 2 distributions.

Calculated $(\text{MinMax(F)Ent})_\text{m}$ and $(\text{MaxMax(F)Ent})_m$, m=1, 2 distributions are given in Table 6.

In Figure 1, the histogram of the wind speed data and the obtained values of $(\text{MinMax(F)Ent})_\text{m}$ and $(\text{MaxMax(F)Ent})_\text{m}$, $m = 1,2$ distributions with two and three moment constraints are demonstrated. It is seen that each of $(\text{MaxMax(F)Ent})_\text{m}$, $m = 1,2$ distributions is more suitable than each of $(\text{MinMax(F)Ent})_\text{m}$, $m = 1, 2$ distributions in terms of modelling data.

4. Conclusion

In this paper, we have suggested the applicability of Generalized Maximum Fuzzy Entropy Methods (GMax(F)EntM) on wind speed data subject to some moment constraints. It should be noted that for the first time distributions of $(\text{MinMax(F)Ent})_\text{m}$, $(\text{MaxMax(F)Ent})_\text{m}$ generated by GMax(F)EntM are applied to the wind energy field.

Table 1 Calculated maximum fuzzy entropy values subject to two moment functions

Moment Functions	Fuzzy Entropy
$1, \sqrt{x}$	**3.3698**
$1, lnx$	3.1008
$1, ln(1+x)$	**2.5656**
$1, ln(1+x^2)$	3.1160

Table 2 Calculated maximum fuzzy entropy values subject to three moment functions

Moment Functions	Fuzzy Entropy
$1, \sqrt{x}, lnx$	**0.3744**
$1, \sqrt{x}, ln(1+x)$	**2.5473**
$1, \sqrt{x}, ln(1+x^2)$	1.1207
$1, lnx, ln(1+x)$	0.7500
$1, lnx, ln(1+x^2)$	2.1394
$1, ln(1+x), ln(1+x^2)$	1.7731

Table 4 The obtained results for $(\textbf{MinMax(F)Ent})_\textbf{m}$, $m = 1, 2$

Distributions of MinMax(F)Ent	Moment Constraints	H	χ^2	RMSE
$(MinMax(F)Ent)_1$	$1, ln(1+x)$	2.5656	0.0025	0.0447
$(MinMax(F)Ent)_2$	$1, \sqrt{x}, lnx$	0.3744	0.0040	0.0563

Table 5 The obtained results for $(\textbf{MaxMax(F)Ent})_\textbf{m}$, $m = 1, 2$

Distributions of MaxMax(F)Ent	Moment Constraints	H	χ^2	RMSE
$(MaxMax(F)Ent)_1$	$1, \sqrt{x}$	3.3698	0.0023	0.0428
$(MaxMax(F)Ent)_2$	$1, \sqrt{x}, ln(1+x)$	2.5473	0.0024	0.0436

Table 6 Membership function values for measured data in December, Distributions of $(MinMax(F)Ent)_m$ and $(MaxMax(F)Ent)_m, m = 1, 2$

V(m/s)	Dec.(h)	μ_i	$(MinMax(F)Ent)_1$	$(MinMax(F)Ent)_2$	$(MaxMax(F)Ent)_1$	$(MaxMax(F)Ent)_1$
0-1	47	0.0672	0.1303	0.0125	0.1585	0.1183
1-2	40	0.0572	0.0688	0.0060	0.0985	0.0644
2-3	38	0.0544	0.0444	0.0039	0.0699	0.0425
3-4	57	0.0815	0.0318	0.0029	0.0525	0.0310
4-5	57	0.0815	0.0242	0.0023	0.0409	0.0240
5-6	48	0.0687	0.0193	0.0018	0.0326	0.0195
6-7	62	0.0887	0.0159	0.0015	0.0265	0.0162
7-8	49	0.0701	0.0134	0.0013	0.0218	0.0138
8-9	41	0.0587	0.0115	0.0011	0.0182	0.0120
9-10	54	0.0773	0.0101	0.0010	0.0153	0.0106
10-11	48	0.0687	0.0089	0.0008	0.0130	0.0094
11-12	39	0.0558	0.0079	0.0007	0.0111	0.0085
12-13	35	0.0501	0.0071	0.0007	0.0096	0.0077
13-14	33	0.0472	0.0065	0.0006	0.0083	0.0070
14-15	20	0.0286	0.0059	0.0005	0.0072	0.0064
15-16	18	0.0258	0.0054	0.0005	0.0063	0.0059
16-17	6	0.0086	0.0050	0.0004	0.0055	0.0055
17-18	4	0.0057	0.0046	0.0004	0.0049	0.0051
18-19	3	0.0043	0.0043	0.0004	0.0043	0.0048

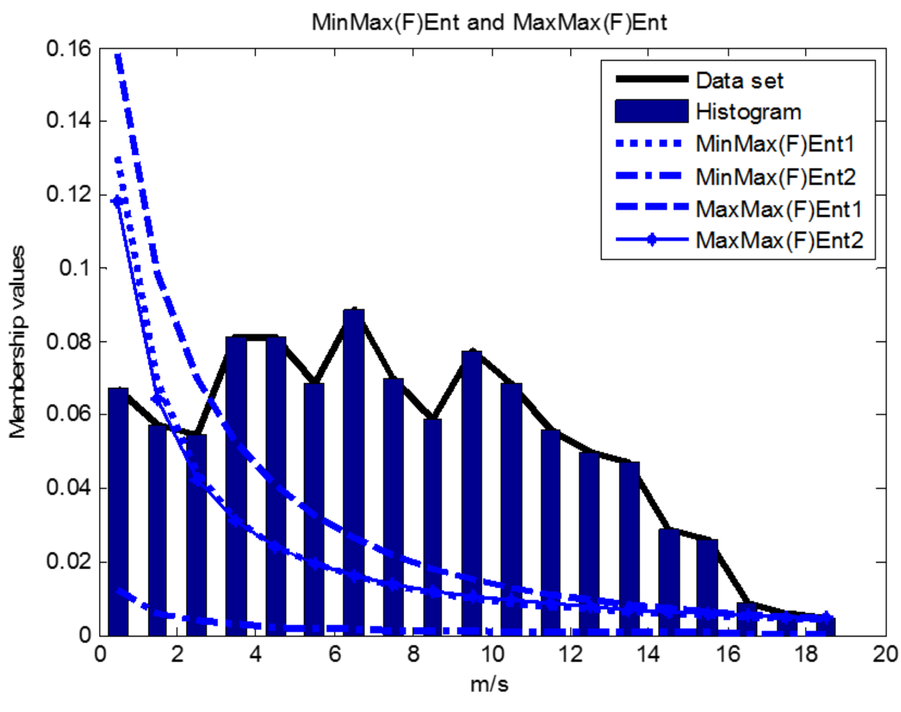

Fig. 1 Histogram of wind speed data and obtained values of $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$ distributions

According to obtained results, for wind speed data, $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$ distributions are compared with respect to different criterias in terms of modellig data. It is shown that the obtained $(\mathrm{MaxMax(F)Ent})_m$ distributions are more suitable in modelling wind speed data than the $(\mathrm{MinMax(F)Ent})_m$ distributions in the sense of RMSE and χ^2 criterias. It is found that $(\mathrm{MaxMax(F)Ent})_m$ distributions can provide better results for the wind speed data. These distributions can be used for evaluation of the wind energy potential. Consequently, the present study gives different and useful results for fuzzy data analysis.

References

[1] J.N. Kapur, H.K. Kesavan, Entropy Optimization Principles with Applications, Academic Press, New York, 1992.

[2] C.E.Shannon, A mathematical theory of communications. Bell System Technical Journal 27 (1948) 379–623.

[3] M. Qing and T. Li ,"Some Properties and New Formula of Fuzzy Entropy", Proceedings of the 2004 IEEE International Conference on Networking, Sensing & Control Taipei, Taiwm, pp. 21-23, 2004.

[4] L. Zadeh, "Probability measures of fuzzy events," Journal of Mathematical Analysisand Applications, vol. 23, pp. 421-427, 1968

[5] A. De Luca and S. Termini, A definition of non-probabilistic entropy in setting of fuzzy set theory, Information and Control 20 (1971) 301-312.

[6] A. Kaufmann, Introduction to the Theory of Fuzzy Subsets, vol. I, Academic Press, New York, 1975.

[7] J. Kopfmacher, On measure of fuzziness, Journal of Mathematical Analysis and Applications, 49 (1975)pp. 529-534.

[8] R. Yager, "On measures of fuzziness and negation, Part I: Membership in the unit interval," International Journal of General Systems, vol. 5, pp. 221-229, 1979.

[9] B. Kosko, "Fuzzy entropy and conditioning," Information Sciences, vol. 40, pp. 165-174, 1986.

[10] D. Bhandari, and N.R. Pal, Some new information measures for fuzzy sets. Information Sciences 67 (1993) 209-228.

[11] N. Pal and K. Pal, "Object background segmentation using a new definition of entropy," IEE Proc. E, vol. 136, pp. 284-295, 1989.

[12] O. Parkash, P.K. Sharma and R. Mahajan, New measures of weighted fuzzy entropy and their applications for the study of maximum weighted fuzzy entropy principle, Information Sciences,178,(2008)2389-2395.

[13] X. Z. Guo and X. L. Xin," Some new generalized entropy formulas of fuzzy sets", J. Nortwest University, vol. 36, pp. 529-532, 2006.

[14] A. Shamilov, Entropy, Information and Entropy Optimization, Turkey, 2009.

[15] A. Shamilov, A development of entropy optimization methods, WSEAS Trans. Math. 5 (2006) pp.-568–575.

[16] A. Shamilov, Generalized entropy optimization problems and the existence of their solutions, Phys. A: Stat. Mech. Appl. 382 (2007) 465–472.

[17] A. Shamilov, Generalized entropy optimization problems with finite moment function sets, Journal of Statistics and Management Systems, 13 (2010) 595-603.

[18] A. Genc and et al., "Estimation of wind power using weibull distribution", Energy Sources, vol. 27, no. 9, pp. 809-822, 2005.

[19] A. Shamilov, S. Ozdemir, N. Yilmaz, Generalized Entropy Optimization Methods for Survival Data, ALT2014: 5th International Conference on Accelerated Life Testing and Degradation Models, Pau, France, 2014, pp.174-183.

[20] Shamilov A., Senturk S.,Yilmaz N.," Generalized Fuzzy Entropy Optimization Methods with application on Wind Speed Data",COIA-2015: The 5th International Conference on Control and Optimization with Industrial Applications, pp. 217-221, Baku, Azerbaijan,2015.