Examples of Cable-Bar Modular Structures Based on the Class-Theta Tensegrity Systems

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Abstract: The relationship between forms and forces is one of the main topics of structural morphology. This harmonious coexisting link is very strong for systems in tensegrity state, commonly called “tensegrity systems”. It is currently apparent that, among the tensegrity systems, there also exist cable-bar cells with a discontinuous network of cables. It is possible to design a separate set of cables inside the cable-bar elementary cell and to establish a self-stress state of equilibrium. In this connection, the author of this paper suggested to assume a new Class-Theta tensegrity systems. Each of the basic tensegrity systems termed Class-Theta possesses an external and internal set of tension components. The shape of Greek capital letter Θ (Theta) reflects two sets of such components (two sets of tendons, cables, etc.). This notation corresponds to Skelton’s Class-κ tensegrity structure. As shown in this paper, the Class-Theta tensegrity cell can exemplify a geometrically and practically useful form for the lightweight and long-span modular structures, mainly but not only in view of civil engineering and architecture.

Key words: Cable-bar, geometry, grid, module, prism, tetrahedron, tensegrity.

1. Introduction

Tensegrity systems are constructed by weak and generally more numerous members (cables) that are flexible in unstressed state, together with strong and local members (bars), but they exhibit sufficient capability of resisting external loads while suitably prestressed. Even if a moderate deforming force is applied at one point of the system, only a transient change is affected in the global form, after which the system once again returns to its equilibrium configuration.

The particular feature of so-called “pure” tensegrity is that bars are never connected to each other, while all cables constitute a connected set, as shown in Fig. 1. This is only one of possible choice of tensegrity systems. Tensegrity is now applicable to architecture as an established structural principle, while it can be applied to other fields as well [1-6]. The problem of form-finding is central in the study of tensegrity systems [5].

The first attempts to create new elementary cells, that are the model tensegrity configurations called systems, were based on some simple characteristics. Tensegrity systems, generally speaking—tensegrity structures, are based on the combination of a few simple but subtle and deep design patterns:

- loading elements remain only in pure tension (in the case of cables) or pure compression (in the case of bars), meaning the structure will only fail if the cables yield or the bars buckle;
- the prestress or the preload in other words, which allows cables to be rigid in tension;
- mechanical stability, which allows the elements to remain in tension or compression as stress on the system increases.

Thanks to these patterns, no structural element experiences a bending moment. This can produce exceptionally rigid structures for their mass and for the cross section of the members.

These structures have another unique property. They can be either flexible/soft or rigid. When the tensegrity structure needs to be rigid and needs sustain...
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Fig. 1 Some examples of the “pure” tensegrity systems, in which the cables constitute a connected set.

Fig. 2 Some examples of the Class-Θ tensegrity systems with separable bars, i.e., “pure” tensegrity systems: (a) a tetrahedron; (b) a triangular prism.

2. Class-Theta Tensegrity Systems

There exists a large literature on the tensegrity systems and structures, which employ a continuous network of tension components.

But currently, it is apparent that, among the cable-bar systems, there also exist the model tensegrity configurations with a discontinuous network of tension components [7-9]. Each of the basic cable-bar configurations possesses an external and internal set of tendons, cables, strings, etc. Therefore, the author suggested to assume a new Class-Theta tensegrity systems. The shape of Greek capital letter Θ (theta) simulates two sets of such components. The notation applied below corresponds to Skelton’s Class-k tensegrity structure [4].

For example, Fig. 2a shows the simplest Class-Θ tensegrity structure, composed of four bars and 10 cables in tension. We can see that the one set of four cables is located inside the other tetrahedral cable network. Moreover, both cable components are separable and interact with a discontinuous set of compressive members to form a stable cable-bar unit in space. In this way, we can build another the Class-Θ “pure” tensegrity units. Among examples, there is a triangular prism shown in Fig. 2b. We can modify an external shape of the “pure” tensegrity units by expansion of the internal set of tension and compression components.

2.1 Class-Theta “Pure” Tensegrity Tetrahedron

There are a large number of possible topologies (node connectivities), with which tensegrity structures can be built. We restrict this part of paper to a

A flexible tensegrity necessarily has a smooth motion that is not a rigid motion of the whole tensegrity structure.
particular minimal form of the Class-Theta “pure” tensegrity cell (thus $\Theta = 1$, and present notation corresponds to Skelton’s Class $k = 1$ tensegrity structure), in which three cables and one bar meet at each exterior node, while two cables and one bar meet at each interior node. This tetrahedral tensegrity system with the connectivity depicted in Fig. 3 has six exterior cables, four interior cables and four bars. Intriguing characteristics of such a tetrahedron include the fact that it transforms from a compact bundle of bars into a full 3D framework as the last cable (either exterior or interior) is pulled in tension, which is form-finding structure. The cable-bar structure becomes a prestressed system.

From an engineering perspective, the Class $\Theta = 1$ tensegrity also possesses unique properties which we note at the outset. The outward appearance of tetrahedral tensegrity cell is based on the space-filling tetrahedron called “T2”, as shown in Fig. 4, and interest readers may consult [10]. This solid is in fact Sommerville’s Type II tetrahedron.

The tetrahedral tensegrity cell, in other words, the tensegrity module, is also a space-filling structure that can be repeated indefinitely by adding module after module in all directions. The symmetry properties of the tetrahedral module can be easy in use during any estimations and precision structural analysis.

To begin with, there are two configurations of four bars inside the tetrahedral cell: the counter-clockwise configuration and its mirror image—the clockwise configuration. It is shown in Fig. 5.

And secondly, each of configurations possesses three two-fold axes of symmetry. Furthermore, the axes are mutually perpendicular to one another and characteristic of whole tetrahedral module. This feature is shown in Fig. 6.

Fig. 7 exemplifies a self-stress state of equilibrium. Each element of the intricate cable-bar structure must be in a stable self-equilibrium.

The tensegrity module shown in Fig. 8 is an example of coplanar configuration, in which an internal nodes occupy the appropriate faces of tetrahedral cell. It is worth to stress that the symmetry
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of tetrahedral module and the outside measurements stayed the same. The length of bars and internal cables suitably rose. As shown in Fig. 9, it is hypothetical possible to build up the coplanar configuration toward a few expanded configurations.

In similar way, we can build another Class-Θ “pure” tensegrity units. Among examples, there is an internal structure of tensegrity tetrahedron shown in Figs. 10a-10c and the mother system from Fig. 2a reinforced by four additional cables as shown in Fig. 10d. In the latter case, the Class $\Theta = 1$ turns into the Class $k = 1$.

2.2 Class-Theta Girders and Grids

The tensegrity girders and grids can be composed

Fig. 5 An enantiomorphism of the Class $\Theta = 1$ tetrahedral cell: (a) the sinistrorse layout of bars; (b) the dextrorse layout of bars.

Fig. 6 Symmetry of the tetrahedral module: (a) the perspective view of structure and its two-fold axes of symmetry; (b) $2x$; (c) $2y$; (d) $2z$ top views.

Fig. 7 A pictorial diagrams of the equilibrium of internal forces for any node of the tetrahedral module: (a) the perspective view of structure; (b) a spatial concurrent force system in the external node; (c) a coplanar concurrent force system in the internal node.

Fig. 8 A previous configuration and coplanar configuration.

Fig. 9 A coplanar configuration and example of expanded configuration.
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Fig. 10 An examples of modification of an internal structure of the Class-Θ tensegrity tetrahedron with separable bars: (a) the perspective view of structure; (b) the tetrahedral configuration of internal connections; (c) the tetrahedral expanded configuration of internal connections; (d) example of the system reinforced by four additional cables [7, 11].

...of the same tetrahedral tensegrity modules and subsequently jointed together in many directions and by several methods.

Some expressive examples of the connecting of Class-Theta tetrahedral tensegrity modules by face-to-face and edge-to-edge methods are shown in Figs. 11-14.

2.3 A Typical Pattern of Modular Truss Based on the Space-Filling Tetrahedron

The space-filling tetrahedral cell shown in Fig. 4 has six exterior cables, four interior cables and four bars. However, it is necessary to add that, in grid suitable for truss structure, only the bars can be used in place of aforementioned exterior cables. All interior cables and bars can omit now. In consequence, we obtain an intriguing form of planar and space trusses.

The simple version of planar truss based on the space-filling tetrahedron T2 and erected by face-to-face linkage is shown in Fig. 15.

Fig. 11 Some variants of the face-to-face linkage.

3. Class-Theta Tensegrity Triangular Prism

There are further examples of the Class-Theta cable-bar configurations. In similar way, we can build a various kinds of prismatic cells.
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Fig. 12  A certain variant of the edge-to-edge linkage. The angle $\alpha$ depends on established length of bars and “internal” cables.

Fig. 13  Some variants of the modular tensegrity girder.

Fig. 14  Example of modular tensegrity grid.
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Fig. 15 Example of modular truss geometry based on the space-filling tetrahedron T2.

Fig. 16 Front view and top view of the physical model: (a) in the mother configuration; (b) in the expanded configuration.

At present, the outward appearance of the mother Class-Theta tensegrity module is based on the triangular prism. A physical model of this module is shown in Fig. 16a. The identical topology of a physical model in expanded configuration is shown in Fig. 16b. However, a definitely different geometry of both modules is visible.

3.1 Geometrical Behaviours

There are two configurations of six bars inside the module: the counter-clockwise configuration and its mirror image—the clockwise configuration. They are shown in Figs. 17a and 17b, respectively.

Each configuration possesses one three-fold and three two-fold axes of symmetry. These features are shown in Figs. 18 and 19. The traits of symmetry are identical in case of the expanded configuration shown in Figs. 16b and 20.

3.2 Examples of Multistage Towers and Grids

Both the Class-Θ “pure” tensegrity triangular prisms in the mother configuration and expanded configuration can exemplify the structural module of

Fig. 17 An enantiomorphism of the Class Θ = 1 prismatic cell.
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Fig. 18 Symmetry of the Class $\Theta = 1$ prismatic cell: (a) the perspective view of cables (on left), all structures (on right) and one three-fold axis of symmetry; (b) three two-fold axes of symmetry in the perspective and top views.

Fig. 19 Symmetry of the Class $\Theta = 1$ prismatic cell: the view in direction of optional two-fold axis of revolution.

towers, girders, grids and more complex spatial tensegrity structures. Some examples of various geometrical solutions for lightweight cable-bar structures are shown in Figs. 21-28.

Fig. 20 The Class-$\Theta$ tensegrity triangular prism in expanded configuration: (a) perspective view; (b) front view and top view.

Fig. 21 An examples of modular tensegrity tower erected by face-to-face linkage: (a) alternate modular structure; (b) homogeneous modular structure; (c) alternate structure with core cables (in yellow); (d) homogeneous structure with core cables.
According to the method applied in all examples for tensegrity towers and which is called “face to face”, adjoining units are connected in the linear self-rigid cable-bar structure. More rigid structures can be obtained with the help of the additional core cables attached to the appropriate internal nodes (Fig. 21c and Fig. 21d).

Both geometry of the triangular prism, like module, and geometry of the modular tower and tension in all cables are easy to maintain, to change or to reinforce, by shortening only the core cables.

Under similar circumstances can erect the modular tensegrity towers with the help of the expanded configuration of mother module.

The simple version of planar tensegrity grid erected by edge-to-edge linkage is shown in Fig. 22. This one was composed of the expanded tensegrity modules. The pattern of common cables and perspective view for this modular triplet can be seen in Figs. 23 and 24, respectively.

The Class-Theta reinforced tensegrity grid made of the same “pure” tensegrity prismatic modules in expanded configuration is shown in Fig. 26. However, the pattern layout of a basic and additional cables can be seen in Fig. 25. By skilful shortening, only the additional cables are possible to maintain or to reinforce of the structural rigidity of a such modular tensegrity grids.

The next modular tensegrity design represents an another geometrical solution of the planar grid based on the Class-Theta “pure” tensegrity triangular prism in expanded configuration. This time, the mirror pairs of basic module are jointed together forming a very original pattern of the planar reinforced grid, as shown in Figs. 27 and 28.
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Fig. 26 The Class-Theta reinforced tensegrity grid: (a) the planar triplet by edge-to-edge linkage in a perspective view completed with the additional cables of top (in blue and dashed lines) and bottom (in red and dashed lines) layers; (b) the layout of more complex planar tensegrity grid made of the prismatic modules in expanded configuration and reinforced by two sets of the additional cables (marked by blue and red dashed lines).

4. Conclusions

Fundamental observations concerning geometrical characteristics of the Class-Theta tensegrity systems ($\Theta = 1$):

- a discontinuous network of cables;
- the length of bars which are significantly smaller in comparison with the longest cables (in case of tensegrity systems with $k = 1$, while this family characteristic is contrary to $\Theta = 1$);
- the system’s outer shapes which can be completely different even though the topology of connections is identical.

Fig. 27 The other version of the Class-Theta reinforced tensegrity grid: (a) the planar four-cell complex made by edge-to-edge linkage in a vertical view, completed with the additional cables of top (in blue and dashed lines) and bottom (in red and dashed lines) layers; (b) the layout of more complex planar tensegrity grid made of the mirror pairs of prismatic module in expanded configuration and then reinforced by two sets of the additional cables (marked by blue and red dashed lines).

Fig. 28 The planar pattern of a basic and additional cables in the modular tensegrity grid.
Moreover, the classic properties of tensegrity systems are still current:

- Isolated bars in compression are situated inside a net of cables, in such a way that the compressed members (bars) do not touch each other and the prestressed tensioned members (cables) delineate the system spatially;
- Loading members are only subjected to the pure compression or pure tension, meaning the structure will only fail if the cables yield or the bars buckle;
- Preload or tensional prestress allows a cables to be rigid in tension;
- No structural member experiences a bending moment.

References


