

# A New Statistical Tool: Scalar Score Function

Zdeněk Fabián

*Institute of Computer Science, Academy of Sciences CR, Pod Vodárenskou věží 2, Prague 182 00, Czech Republic*

Received: October 19, 2010 / Accepted: January 06, 2011 / Published: February 25, 2011.

**Abstract:** The basic inference function of mathematical statistics, the score function, is a vector function. The author has introduced the scalar score, a scalar inference function, which reflects main features of a continuous probability distribution and which is simple. Its simplicity makes it possible to introduce new relevant numerical characteristics of continuous distributions. The t-mean and score variance are descriptions of distributions without the drawbacks of the mean and variance, which may not exist even in cases of regular distributions. Their sample counterparts appear to be alternative descriptions of the observed data. The scalar score itself appears to be a new mathematical tool, which could be used in solving traditional statistical problems for models far from the normal one, skewed and heavy-tailed.

**Key words:** Statistics, inference function, data characteristics, point estimates, heavy tails.

## 1. Introduction

In statistical estimation problems, estimates are obtained as the solutions to equations of the form

$$\Psi(\text{data}; \theta) = 0$$

where  $\theta = (\theta_1, \dots, \theta_m)$  is a vector of parameters of interest and  $\Psi$  is an inference function, the meaning of which is to adapt the data to the assumed model.

A parametric model is a parametric family of distributions  $F_\theta$  with probability densities  $f(x; \theta)$ . The commonly used inference function is the score function, the vector  $\Psi = (\Psi_{\theta_1}, \dots, \Psi_{\theta_m})$  of derivatives of  $\log f(x; \theta)$  with respect to the components of  $\theta$ . The data  $x_1, \dots, x_n$  are considered as realizations of random variables  $X_1, \dots, X_n$ , independent and identically distributed according to  $F_{\theta_0}$ , a member of the assumed model  $F_\theta$ . The solution of the system of equations

$$\sum_{i=1}^n \Psi_{\theta_k}(x_i; \theta) = 0, \quad k = 1, \dots, m,$$

is the maximum likelihood estimate  $\hat{\theta}_{ML}$  of  $\theta_0$ . Since  $\hat{\theta}_{ML}$  has lowest possible variance, density  $f(x; \hat{\theta}_{ML})$  is considered as the best result of the

estimation process [1].

Apart from the fact that  $\hat{\theta}_{ML}$  is influenced by observations far from the bulk of the data (outliers: this problem is solved by robust statistics), this result has some other, usually not accented drawbacks:

(1) Parameters of different models are, as a rule, of different nature (location, scale, shape, frailty). A comparison of the precision (variance) of estimates in differently parametrized models is a difficult task;

(2) Instead of  $f(x; \hat{\theta}_{ML})$ , a few numbers characterizing the data would be often more useful in further analysis. A desirable description of the data seems to be something like “center” and “radius”, perhaps skewness of the data. The commonly used numerical characteristic of distributions are the mean  $m = EX = \int xf(x) dx$ , variance  $m_2 = E(X - m)^2$  and higher central moments  $m_k = E(X - EX)^k$ ; it seems that  $\hat{m} = m(\hat{\theta}_{ML})$  and  $\hat{m}_k = m_k(\hat{\theta}_{ML})$  should

be the values we search for. However, such an approach is not used in statistical practice. Moments are often queer expressions containing special functions, and moments of heavy-tailed distributions (distributions whose probability densities are

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**Corresponding author:** Zdeněk Fabián, CSc., research field: statistic estimation. E-mail: zdenek@cs.cas.cz.

approaching zero too slowly) do not exist (the corresponding integrals are infinite);

(3) More complex statistical problems, such as estimation of the degree of similarity of random variables  $X$  and  $Y$ , are usually solved by using “pure” data without adapting them to the assumed model.

A well-known example is the measure of association of two random variables, the Pearson correlation coefficient:

$$\rho(X, Y) = \frac{1}{n} \sum_{i=1}^n (x_i - m_X)(y_i - m_Y)$$

which is not able to make clear which part of the dependence stems from the real dependence and which part stems from the properties of marginal distributions of  $X$  and  $Y$ .

The cause is that the score function is a vector function, suitable for estimation of parameters, but too complicated to afford useful proposals of sensible numeric characteristics of distributions and too complicated to be used in more complex problems.

In this paper, we describe a scalar inference function proposed by the author [2-5]. It reflects basic features of the model distribution, being simultaneously a simple scalar function. We outline new possibilities of description of probability distributions and data taken from them, and suggest possible use of the function for solution of traditional statistical problems, which appears to be particularly useful if the underlying distribution is skewed and/or heavy-tailed.

## 2. Scalar Score

As a scalar score of random variable  $X$  with distribution  $G$  and differentiable density  $g$  positive on the whole real line  $R$  was identified, using lesson drawn from Hampel et. al. [6], function describing the relative change of the density:

$$S_G(x) = -\frac{g'(x)}{g(x)} \tag{1}$$

The reason is that if  $G$  as a symbol is a location distribution (that is, if it is in the form  $G(x - \mu)$  where  $\mu \in R$  is the location parameter, indicating position of the maximum of density), function:

$$S_G(x - \mu) = \Psi_{\mu}(x - \mu)$$

equals the score function for  $\mu$ . The solution of equation:

$$S_G(x; \theta) = 0 \tag{2}$$

where  $S_G(x; \theta)$  is a parametric form of Eq. (1), is the coordinate of the maximum of density (mode) even if  $G$  is not a location distribution.

However, Eq. (1) is not a suitable description of distributions with densities positive only on a part  $X \neq R$  of the real line (that is, with support  $X \neq R$ ), since the maximum of the density can lie at the edge of the support and equation  $S_G(x) = 0$  may have no solution. Based on the old idea of Johnson [7], we suggested to view any random variable  $X$  with distribution  $F$  supported by  $X \neq R$  as a transformed random variable  $X = \eta^{-1}(Y)$ , where  $Y = \eta(X)$  has a “prototype” distribution  $G$  with support  $R$ . By using a mapping  $\eta: X \rightarrow R$ , an interesting function of distribution  $F(x) = G(\eta(x))$  was identified the transformed scalar score of the prototype:

$$T(x) = S_G(\eta(x)) \tag{3}$$

Eq. (3) can be expressed by means of the density  $f$  of  $F$  as:

$$T(x) = -\frac{1}{f(x)} \frac{d}{dx} \left( \frac{1}{\eta'(x)} f(x) \right) \tag{4}$$

Eq. (4) expresses the relative change of a “basic component of the density”, which appeared to be the density divided by the Jacobian of the mapping used.

For comparison of properties of functions (4) of different distributions it is necessary to use consistently one concrete mapping for a given support. We used that one providing simple forms of Eq. (3) for a large amount of commonly used distributions. According to Johnson [7], we put

$$\eta(x) = \begin{cases} \log(x - a) & \text{if } X = (a, \infty) \\ \log \frac{x}{1-x} & \text{if } X = (0, 1) \end{cases} \tag{5}$$

(a generalization to common intervals is obvious). The result, Eq. (4) with  $\eta$  given by Eq. (5), is called the

transformation-based score or shortly the t-score. Scalar scores of some “prototypes” and t-scores of the corresponding transformed distributions on  $(0, \infty)$  are given in Fig. 1 and Fig. 2.

A location-type distribution  $G(y - \mu)$  is mapped into  $(0, \infty)$  as distribution:

$$F(x; \tau) = G(\eta(x) - \eta(\tau)) \quad (6)$$

The parameter  $\tau = \eta^{-1}(\mu)$  is the transformed location parameter [8], the “image” on  $X$  of the location of the prototype. It can be taken as a “center” of  $F(x; \tau)$ , as it is, according to Eq. (6) and Eq. (2),

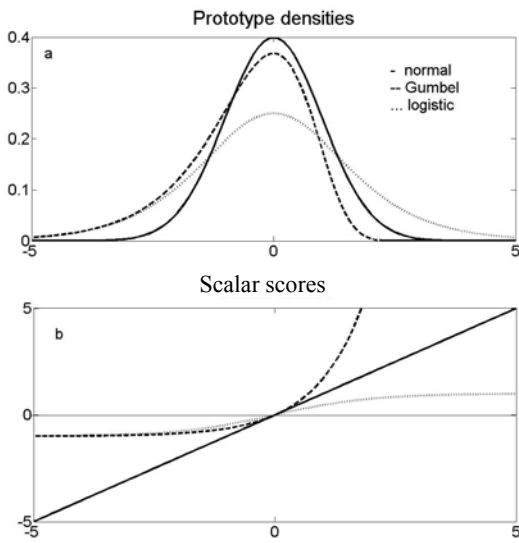


Fig. 1 Prototype distributions. a. densities and b. scalar scores of distributions with support  $R$ .

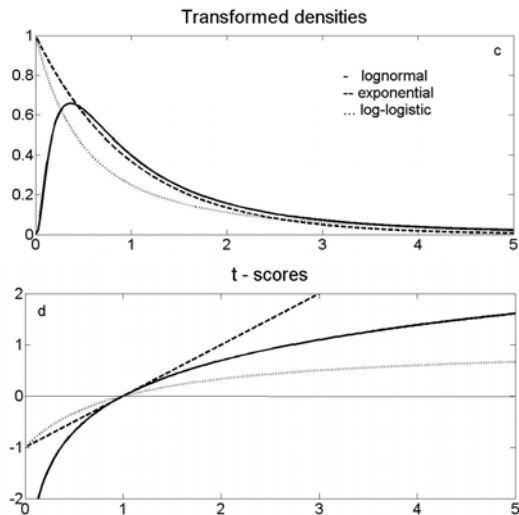


Fig. 2 Transformed distributions. c. densities and d. t-scores of corresponding transformed distributions with support  $(0, \infty)$ .

the solution of equation  $T(x; \tau) = 0$ , where  $T(x; \tau)$  is the corresponding t-score. It was proved [2] for this particular class of distributions that it holds true:

$$\eta'(\tau)T(x; \tau) = \Psi_\tau(x; \tau) \quad (7)$$

We thus obtained a function which equals the score function for parameter expressing the position of distribution  $F$  on the x-axis. The score function  $\Psi_\tau$  is decomposed into a product of two terms: an individual member characterizing the distribution (obtained by differentiating with respect to the variable) and the value of the Jacobian of the transformation at  $\tau$ .

A generalization for arbitrary continuous distribution is the following [3]: As a measure of central tendency of distribution  $F(x; \theta)$  we suggest the zero of its t-score:

$$x^*(\theta): T(x; \theta) = 0$$

called the transformation-based mean or shortly the t-mean (actually, the transformed mode of the prototype). Eq. (7) was generalized by using the t-mean instead of  $\tau$ . The result:

$$S(x; \theta) = \eta'(x^*)T(x; \theta) \quad (8)$$

is called the scalar score (actually the “score function for t-mean”). We suggest  $S(x; \theta)$  as a scalar inference function of distribution  $F$ , either the t-mean is a parameter of the distribution or not. For particular classes of distributions with location and transformed location parameter it is identical with the score function for this parameter. In other cases it is a new function.

### 3. Description of Distributions

Besides the cumulative distribution function  $F(x)$  and probability density  $f(x)$ , probability distribution  $F$  can be described by the scalar score  $S(x) = \eta'(x^*)T(x)$ . Since Eq. (7) is proportional to the influence function of the maximum likelihood estimator of  $\tau$ , the value  $S(x)$  can be analogically understood as a relative influence of  $x \in X$  on an estimate of the t-mean.

If the probability density of prototype distribution goes to  $\pm\infty$  slowly as  $e^{-x}$  (the distribution is

heavy-tailed),  $S_G$  is bounded. Due to Eq. (3), scalar scores of heavy-tailed distributions with arbitrary support are bounded. This property implies the existence of score moments:

$$M_k = ES^k(X) = \int_X S^k(x)f(x) dx \quad (9)$$

which can be used as numeric characteristics of distributions.  $M_1 = 0$  (scalar score is centered around the t-mean). The t-mean exists and is unique for distributions with unimodal prototypes (in other cases, some additional convention is necessary).

The value  $M_2 = ES^2$  of distributions with location and transformed location is, respectively, the Fisher information for the location and transformed location parameter. Accordingly,  $ES^2$  is the Fisher information for the t-mean. The usual regularity conditions secure that it is finite. The reciprocal value:

$$\omega^2 = 1/ES^2 \quad (10)$$

appears to be a natural measure of the variability (dispersion) of the distribution even in cases in which the usual variance does not exist [3]. We call it a score variance. For distributions with support  $(0, \infty)$ , Eq. (10) turns into  $\omega^2 = (x^*)^2 / ET^2$ . T-means and score variances of some distributions are given in Table 1.

Let us remark that  $\gamma = M_3 / M_2^{3/2}$  characterizes skewness; a real skewness for distributions with support  $R$  and skewness with respect to the basic form for distributions supported by  $(0, \infty)$  (which is itself skewed).  $M_4$  characterizes flatness. The analog of Pearson's measure of kurtosis forms a logical structure right opposite to the ordinary kurtosis.

Function  $S^2(x)$  attains its minimum at  $x^*$ , which is the least informative point of the distribution [10], and  $ES^2$  describes information.  $S^2(x)$  can thus be thought of an information function, expressing relative information contained in observation  $x$ .

Let us consider a distance between  $x_1, x_2 \in X$  given by:

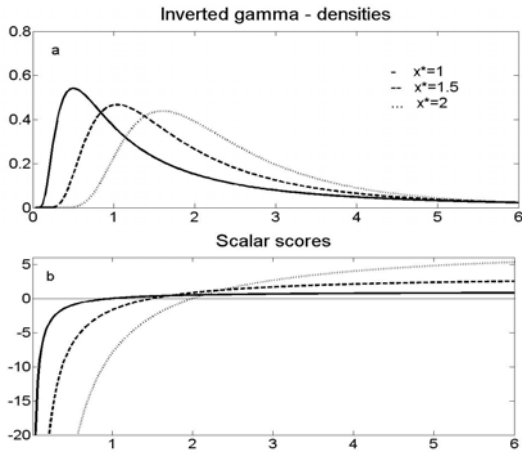
$$d(x_1, x_2) = |S(x_2) - S(x_1)| \quad (11)$$

Parametric forms of Eq. (11) are in fact considered when using maximum likelihood estimators in cases of distributions with location and transformed location parameter. It is apparent that the meaning of Eq. (11) is the distance of relative influences of  $x_1$  and  $x_2$ . For standard normal and standard exponential distributions  $d(x_1, x_2) = |x_2 - x_1|$  is Euclidean. However, for skewed and/or heavy-tailed distributions, distance (11) is nonlinear. As an analogy, the temperature scale is linear. However, the distance between  $-272^\circ\text{C}$  and  $-272.3^\circ\text{C}$ , say, can be expressed in millions dollar and should be large, whereas the distance between  $1,000,000^\circ\text{C}$  and  $1,200,000^\circ\text{C}$  is not interesting and could be small. A nonlinear temperature scale would be for some practical purposes more suitable. A proposed distance between  $x_1, x_2 \in X = (-273.15^\circ\text{C}, \infty)$  could be measured by (11) with  $S$  being a "scalar score function of distribution of temperature", derived by means of Eq. (4) and Eq. (8) with  $f$  describing "a rate of incidence of temperature data in man's life". It could resemble, perhaps, the scalar score of the inverted gamma distribution in Fig. 3.

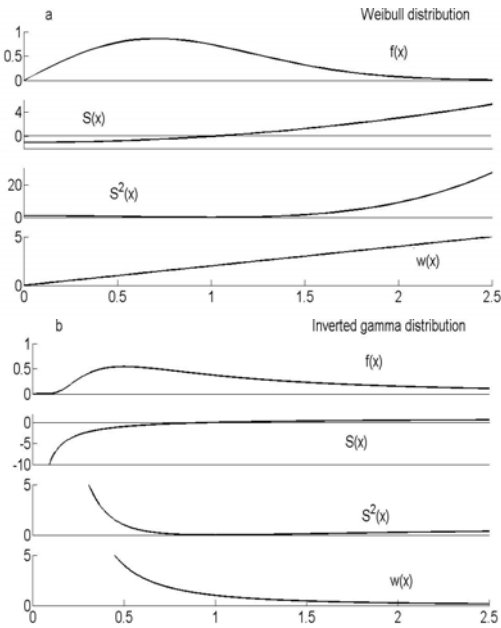
**Table 1 T-mean and score variance of some distributions.**

Distribution	$f(x)$	$T(x)$	$x^*$	$\omega^2$
Weibull	$\frac{c}{\tau} (\frac{x}{\tau})^{c-1} e^{-(x/\tau)^c}$	$c((x/\tau)^c - 1)$	$\tau$	$\tau^2 / c^2$
gamma	$\frac{\gamma^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\gamma x}$	$\gamma x - \alpha$	$\alpha / \gamma$	$\alpha / \gamma^2$
lognormal	$\frac{c}{x\sqrt{2\pi}} e^{-\frac{1}{2} \log^2(x/\tau)^c}$	$c \log(x/\tau)^c$	$\tau$	$\tau^2 / c^2$
Pareto	$c x^{-(c+1)}$	$c - (c+1) / x$	$(c+1) / c$	$(c+2) / c^3$
Lomax	$\alpha / (1+x)^{\alpha+1}$	$(\alpha x - 1) / (x+1)$	$1 / \alpha$	$(\alpha+2) / \alpha^3$
inv. gamma	$\frac{\gamma^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} e^{-\gamma/x}$	$\alpha - \gamma / x$	$\gamma / \alpha$	$\gamma^2 / \alpha^3$

Weibull and lognormal are distributions with transformed location parameter. Pareto, Lomax and inverted gamma are heavy-tailed distributions, having for certain range of parameters neither mean nor variance.



**Fig. 3** Numerical characteristics of distributions. **a.** densities and **b.** scalar scores of inverted gamma distributions with parameters chosen in such a way that t-means are  $x^* = 1, 1.5, 2$  and score variance  $\omega^2 = 1$ . All the plotted distributions have neither mean nor variance. It is apparent that t-mean characterizes a position of distributions on the  $x$  – axis and that all three distributions have a similar character of variability. Scalar scores of inverted gamma distributions are not sensitive to large values, but exhibit large sensitivity to observations near zero.



**Fig. 4** Functions describing distributions. Density, scalar score, information function and weight function of **a.** Weibull distribution ( $c = 2$ ) and **b.** inverted gamma distribution.

Writing Eq. (11) in the form:

$$d(x_1, x_2) = \int_{x_1}^{x_2} dS(x)$$

we obtain another significant function of distribution  $F$ , function  $w(x) = dS(x)/dx$ , which could be, perhaps, explained as the weight function, giving to any  $x \in X$  a relative weight of the observation in the assumed model. Basic functions describing two standard probability distributions are plotted in Fig. 4.

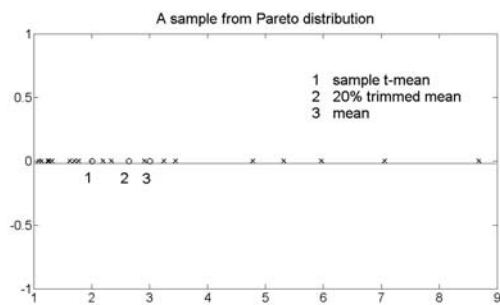
#### 4. Data Characteristics

Given data  $x_1, \dots, x_n$  and model family  $F_\theta$ , new sample characteristics of central tendency (“center”) and dispersion (square of “radius”) can be obtained as functions of estimated parameters: they are the sample t-mean  $\hat{x}_{ML}^* = x^*(\hat{\theta}_{ML})$  and the sample score variance  $\hat{\omega}_{ML}^2 = \omega^2(\hat{\theta}_{ML})$ . They can be alternatively used for a description of data samples, particularly of samples from skewed and/or heavy-tailed distributions, for which the usual characteristics cannot be used (Fig. 5 and Fig. 6). By using them, it is easy to compare results of estimation in different models [4].

Besides the maximum likelihood estimates, one can use the score moment estimates  $\hat{\theta}_{SM}$  as the solution of equations:

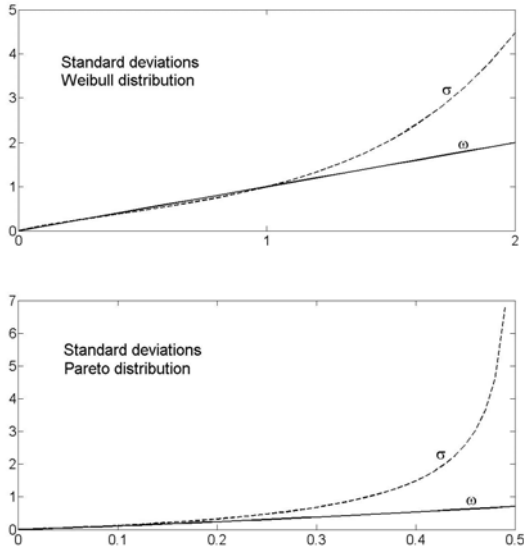
$$\frac{1}{n} \sum_{i=1}^n S^k(x_i; \theta) - E_\theta S^k = 0, \quad k = 1, \dots, m \quad (12)$$

derived from Eq. (9) using the substitution principle.



**Fig. 5** Typical value of a sample. A sample (the one with a small largest value) of length  $n = 20$  was generated from Pareto distribution with  $c = 0.95$ , i.e., from a distribution without mean. The sample mean (3) can be determined but it is theoretically unjustified. The 20% trimmed mean (2) is known to be very robust estimator, however, it does not take into account the properties of distributions. The

sample t-mean (1) characterizes a typical value of the sample.



**Fig. 6 Standard deviations.**  $\sigma$  and score standard deviation  $\omega$  are shown as functions of the reciprocal value of parameter  $c$  of Weibull and Pareto distributions.  $\sigma$  of heavy-tailed Pareto distribution does not exist if  $c \leq 2$ . For large  $c$ ,  $\omega \approx \sigma$ . For  $c$  small, description of variability of both distributions by the standard score deviation  $\omega$  is the only sensible method we know.

The score moments are often expressed by elementary functions of parameters [11]. Since the scalar scores of heavy-tailed distributions are bounded, a large observation  $x_i$  has no decisive influence on estimates since it enters into estimation equations by means of  $S(x_i; \theta)$  only.

In some cases, the first equation of (12) has a form:

$$\hat{x}_{SM}^* : \sum_{i=1}^n S(x_i; x^*) = 0$$

Then the sample score variance of samples from distributions with support  $(0, \infty)$  is given by:

$$\hat{\omega}^2 = \frac{(\hat{x}_{SM}^*)^2}{n^{-1} \sum_{i=1}^n T^2(x_i; \hat{x}_{SM}^*)}$$

Examples are given in Table 2. In a general case, however, the sample characteristics are to be determined as  $\hat{x}^* = x^*(\hat{\theta})$  and  $\hat{\omega}^2 = \omega^2(\hat{\theta})$ , where  $\hat{\theta}$  is either the maximum likelihood or the score moment estimate. The score moment estimates are generally not efficient (that is, they are asymptotically

normal but their asymptotic variances are not the lowest possible, contrary to the maximum likelihood

**Table 2 Sample t-mean and sample score variance of distributions from Table 1.**

Distribution	$\hat{x}^*$	$\hat{\omega}^2$
Exponential	$\bar{x} = n^{-1} \sum x_i$	$\bar{x}^2$
Gamma	$\bar{x}$	$n^{-1} \sum (x_i - \bar{x})^2$
Lognormal	$\bar{x}_G = n^{-1} \prod x_i$	$n^{-1} \sum \log^2 x_i / \bar{x}_G$
Pareto	$\bar{x}_H = n / \sum 1/x_i$	$(2\bar{x}_H - 1)(\bar{x}_H - 1)^2$
Lomax	$\bar{x}_L = \sum \frac{1}{x_i+1} / \sum \frac{x_i}{x_i+1}$	$\bar{x}_L^2 (2\bar{x}_L + 1)$
Inverted Gamma	$\bar{x}_H$	$n\bar{x}_H^2 / \sum (1 - \bar{x}_H/x_i)^2$

The exponential distribution is Weibull distribution with  $c=1$ . The sample t-mean of distributions with linear scalar score is the arithmetic mean,  $\bar{x}_G$  is the geometric mean and  $\bar{x}_H$  harmonic mean. The t-mean of the Lomax distribution (a simple member of the generalized logistic family, see [9]) is given by an original closed form. The sample score variance of one-parameter distributions is a function of the sample t-mean. The sample score variance of gamma distribution is the usual variance. For sample score variances of two-parameter lognormal and inverted gamma distributions we found closed-form formulas. In a general case, both  $\hat{x}^*$  and  $\hat{\omega}^2$  can be determined as functions of estimated of parameters.

estimates), but they are robust in cases of heavy-tailed models [12]. Confidence intervals for  $\hat{x}_{SM}^*$  can be established using a modification of the Rao score test [1] or by the use of the distance  $d(\hat{x}_{SM}^*, x_0)$ , where  $d$  is given by Eq. (11), see Ref. [5].

### 5. Other Applications

The scalar score of a normal distribution is  $S(x; \mu, \sigma) = (x - \mu)/\sigma^2$ , so that the t-mean is the mean and score variance is the variance. For observed data coming from non-normal distributions we obtained new data characteristics. The scalar score can be used as an inference function for solution of traditional statistical problems. We mention three of possible applications:

(1) **Linear regression with non-normal residuals.**

Let  $Y = \alpha_0 + \alpha_1 X + \varepsilon$ , where  $\varepsilon$  is random variable with distribution  $F_\varepsilon$ . For estimation of coefficients  $\alpha_0$  and  $\alpha_1$ , a criterion of minimal Fischer information of residual errors has been used, i.e.:

$$\frac{1}{n} \sum_{i=1}^n S^2(\varepsilon_i) = \min. \quad (13)$$

where  $S$  is the scalar score of  $F_\varepsilon$  and  $\varepsilon_i = y_i - (\alpha_0 + \alpha_1 x_i)$  are the residual errors. For normally distributed residuals, Eq. (13) reduces to the least squares method. For skewed heavy-tailed  $F_\varepsilon$  we obtained results different from those provided by robust regression by using the Huber's (symmetric) inference function (Fig. 7). We conclude that if one can guess the type of parametric family of underlying distribution, the method based on Eq. (13) is the best one;

**(2) Distribution-dependent correlation coefficient.**

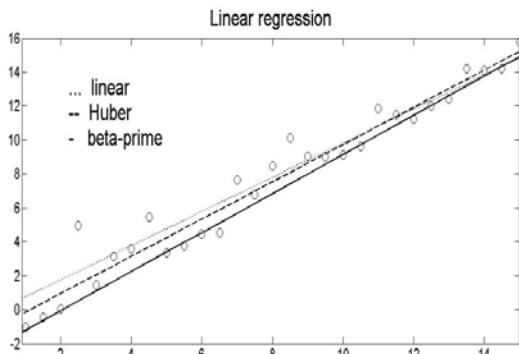
The score correlation coefficient of random variables  $X, Y$  with distributions  $F_X, F_Y$  and scalar scores  $S_X, S_Y$ , respectively, is

$$\rho_s(X, Y) = \frac{E(S_X, S_Y)}{(ES_X^2 ES_Y^2)^{1/2}}$$

It holds that  $-1 \leq \rho_s \leq 1$  and  $\rho_s(X, Y) = 0$  for independent  $X$  and  $Y$ . Simulation experiments have shown that  $\rho_s$  can detect an association of random variables even if they have heavy-tailed distributions, similarly to the Spearman rank correlation coefficients [13] (Fig. 8);

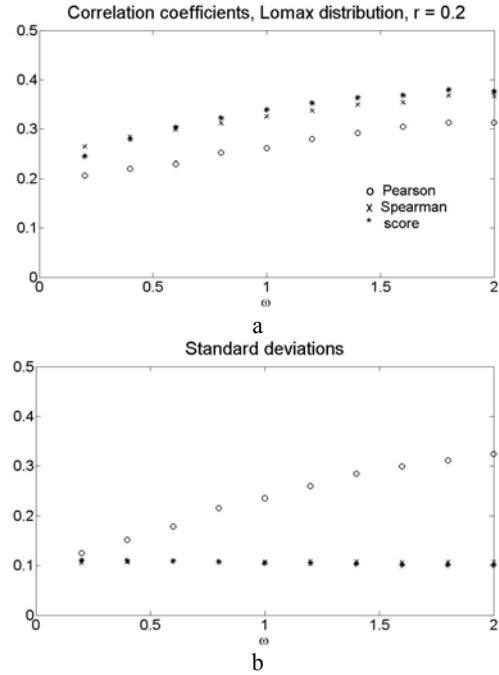
**(3) Spectral properties of heavy-tailed processes.**

If the distribution of random process  $\{X_t\}$  is heavy-tailed, random variable  $X_t$  does not possess finite variance and  $\{X_t\}$  does not possess spectral density. Since the moments of random variable  $S(X_t)$



**Fig. 7 Linear regression.** Comparison of least-squares regression, robust regression and score regression lines for data  $y = -2.3 + 1.2x + \varepsilon$  where  $\varepsilon$  was generated as random variable with Lomax distribution. The score regression line is below the robust regression line since the

scalar score takes into account the non-symmetry of the distribution (and, consequently, of the generated data).



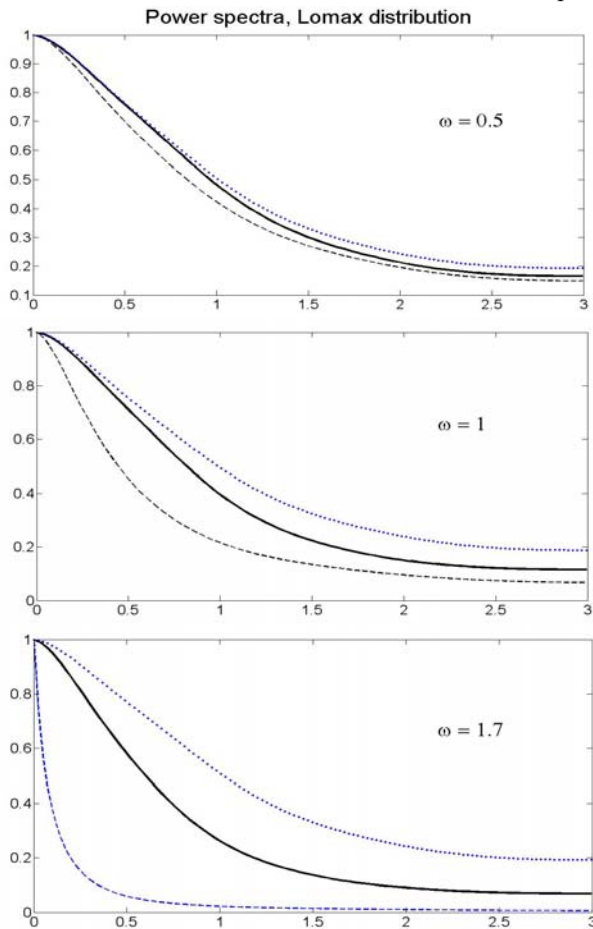
**Fig. 8 Correlation coefficients.** Couples  $(X, Z)$  were generated as independent random samples of length 75 points from the Lomax distribution. The sample correlation coefficients between  $X$  and  $Y = \alpha X + (1 - \alpha)Z$  were estimated with  $\alpha$  such that theoretical value of correlation is  $r = 0.2$ . a. average values of correlation coefficients Pearson (o), score (\*) and Spearman (x) correlation coefficients, and b. their average standard deviations as functions of variability  $\omega$  of the distribution. It is apparent from the latter plot that the usual Pearson correlation coefficient loses for heavy-tailed distribution any meaning. The sample score correlation coefficients are in this particular case closed to the Spearman rank correlation coefficients and, although biased, they can detect an association of random variables with heavy-tailed distributions.

are finite for any fixed  $t$ , one can study spectral densities of process  $\{S(X_t)\}$ . We have found that the score power spectrum of a process with small variability (small  $\omega$ ) is similar to that of the original process, whereas for processes with large variability it seems to be a sensible estimate of the spectral content of  $\{X_t\}$  [14], as illustrated in Fig. 9.

**6. Conclusions**

We described the way to an introduction of a new inference function of mathematical statistics and show

some of its advantages, the main of them is that it enables to introduce relevant characteristics of data samples



**Fig. 9 Power spectra.** Average power spectra of autoregressive signal  $X_t = 0.4X_{t-1} + Z_t$ , where  $Z_t$  is a white noise with Lomax distributions with three different  $\omega$ . Dashed lines: spectra of  $\log X_t$  (the used way of estimating spectra of positive signals), full lines: spectra of  $S(X_t)$ , where  $S$  is the scalar score of the Lomax distribution, dotted line: spectrum of  $X_t$  with standard normal  $Z_t$ . The estimated power spectra are similar for low  $\omega$ . With increasing variability  $\omega$ , the score spectra are still usable, whereas the log-spectra are largely distorted.

taken from arbitrary continuous distribution. Similarly as in any parametric method, the results of processing real data are crucially dependent on the model adopted. The sample t-mean and sample score variance may now make it possible to compare the results of estimation in arbitrarily parametrized models.

## Acknowledgments

The research presented was supported by projects AV0Z10300504 and GACR 205/09/1079.

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