Life Annuities Calculation in Algeria: Continuous Time Approach

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The present paper aims to show the impact of continuous time calculation on life insurance pricing and reserving in the Algerian context. The discrete time approach allows insurance companies to facilitate calculation process but with less accuracy. This approach implies constancy of death quotients during a year. However, the death risk is a continuous function in time. For more accuracy and equity in pricing, calculation needs to consider the exact dates of different payments and also a continuous capitalization process. This gives more adequate premium with fewer hypotheses. This work shows how insurers can propose more adequate pricing using the same actuarial life table.

Keywords: Life annuities, life table, continuous time, fitting, extrapolating, Algeria.

Introduction

Life insurance allows all individual to secure himself or his family members against specific risks. For insurers, the inquietude of the individuals represents an opportunity to take benefits if they accept premiums paid by individuals and engages to pay them the contracted indemnity. To better manage his engagement, the insurer must better estimate the supported risk especially for long period contracts. Principally two parameters must be taken in calculation: interest rate and life table.

In practice, a series of contributions are paid during the period of activity, entitling the insured to see an annuity generally taking effect with the age of retirement until death. In such logic, segmentation automatically arises between the phase of capital formation and phase of the annuity payments. The transition from one phase to the other undergoes the principle of equivalence between subscriptions and pensions. This equivalence is based on two factors: a life factor and other financial. The first factor referred to the fact that the terms of the annuity shall be paid only for the survivors to the date of payment, which involves the survival function of the insured. The second factor is related to the investment of contributions and which involves the use of a technical interest rate defined as a security measure, below the rate of investment in the market.

For the case of the Algerian insurance market, the law 06-04 of 20th February 2006 obliged to separate between life and non-life activities. Because of this decision, all companies might take in equilibrium their activities superlatively in life and non-life branches. So, life insurers were obliged to be more efficient in premiums / Reserves calculation. By the amplification of the number of life insures companies in the Algerian markets these last years, the challenge is to propose adequate contract while controlling the insured risk.

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Discrete time calculation allows facilitating calculation process by using some hypotheses. If the financial stability hypotheses can be kept given the relative stability of the interest rate in the Algerian market, the lifetime hypothesis appears too limiting. For purposes of simplifying calculations, pricing and provisioning, pensioners are grouped by age and supposed to assume the same mortality risk. In other words, an individual that is born at the beginning of the year is supposed to have the same probability of death as the one who is born at the end of the same year.

The adoption of these hypotheses may result to overvalue the real risk represented by the insured group, and therefore by the annuitants portfolio. These results in the amplification of the estimation of the future commitment of the insurer and therefore error of the mathematical reserve the insurer assumes enable it to fulfill the commitment to future pensioners. And it is in this context that fits this work, which will be on the table and the calculation method used for provisioning annuities.

The calculation time content, based on an instantaneous rate, (death / interest) modeling, seems to result in an estimate closer to the reality of mathematical calculation that discrete calculation method.

Life insurance calculation is based on two essential parameters: interest rate and life table. With continuous approach, these parameters should be reformulated in continuous form. If "i" indicates the annual interest rate used, the instantaneous interest rate is given by $r = \ln(1 + i)$. Similarly, mortality law replaces life table. To do this, several mortality models can be used: Makeham, Gompertz, Exponential…etc. the choice of the adequate model is obtained by the Least Square Errors between table values and model values. For mortality in old ages, specific models are used. The mortality changes trend beyond specific age, usually 80 or 85 (Caole & Gou, 1990). Numerous models can be implemented for (Quashie & Denuit, 2005). The Data used in this paper is the Algerian life tables (Female) published by the National Statistics Office for the years 2010, 2011and 2012. All these tables are extended to the age of 85. In first, average table is constructed. The second step is moving from the five-year table to the single year table by using interpolating techniques [Lagrange, Karup-King]. Then, we use fitting mortality models to estimate the parameters of the mortality function. Beyond the age of 80, we have to calculate the adequate function.

Comparison between the two calculation methods must be done using common data. For this, it is preferable to begin with updating the life table used for this kind of calculation. We remind that by security pricing measures, female tables are used for life insurance calculation, and male tables for Death-insurance calculation.

Life Table Construction

Average life table 2010-2012

By the present work, we will try to show the impact of continuous calculation on annuities provisioning. In first, we will try to construct a mortality table updated on recent data, i.e. female mortality data for the period 2010-2011-2012. The use of three years period is explained by two arguments:

• The use of multi years average table allows to eliminate annuals perturbations on mortality estimated data;

• Also, we can use more than three years for improving table quality. In terms of Algerian life table closure, only the three recent tables were expanded until at the age of 85 years. So, the use of only these three tables allows having a longer age range.

The published life tables for the Algerian female population for the years 2010, 2011 and 2012 are represented in figure 1.
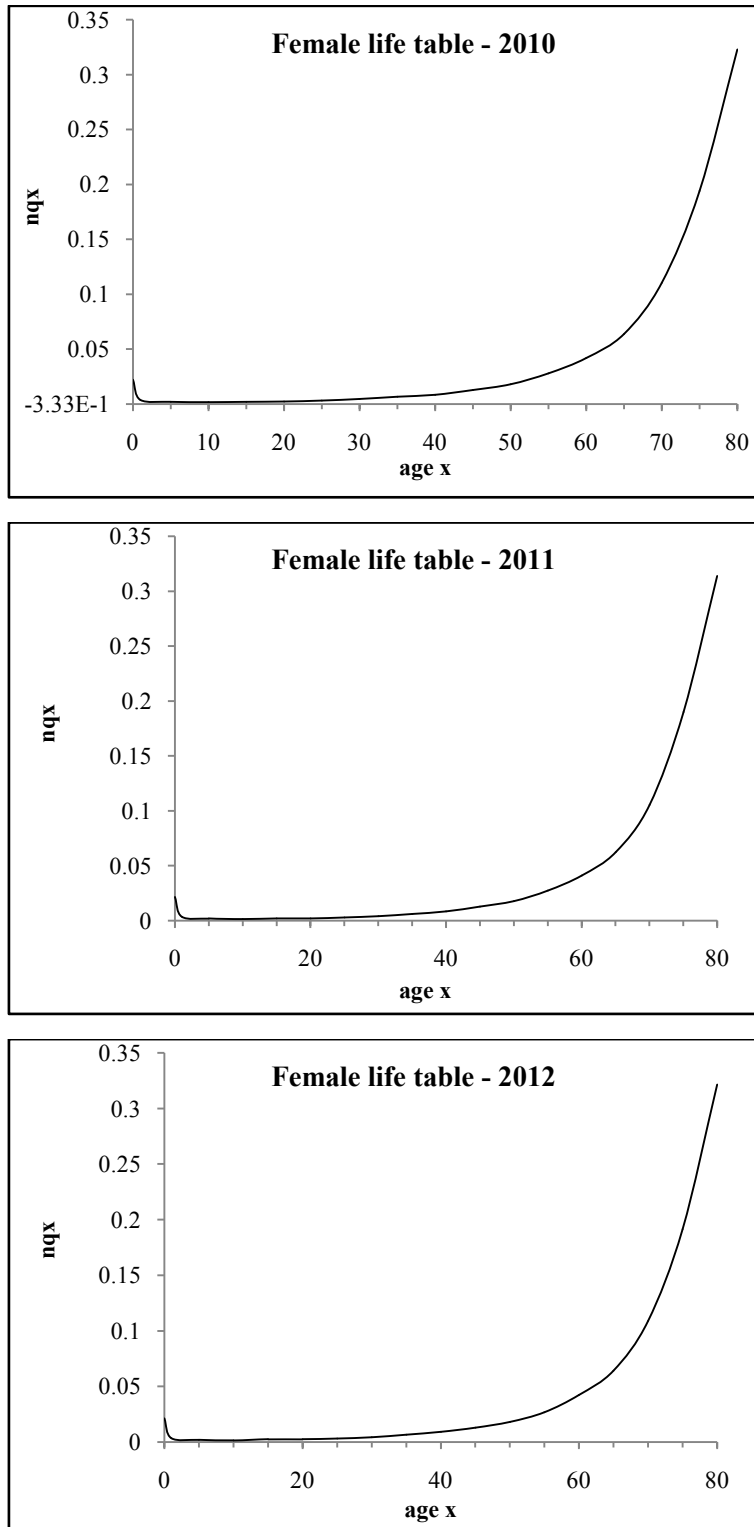
Figure 1. Official Algerian life tables – Female Population (2010-2012).

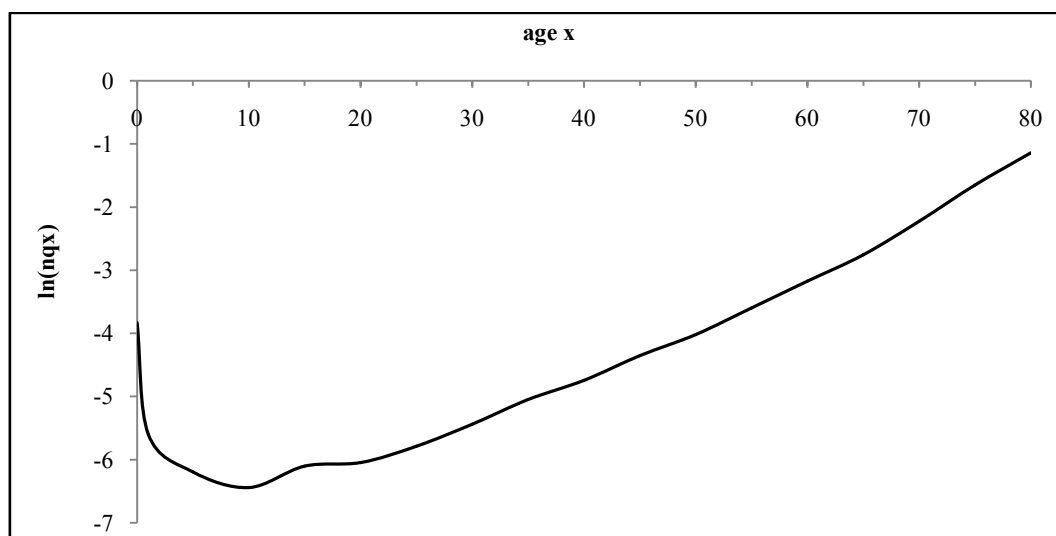Average life table for the period 2010 – 2012, expressed on ln(nqx) is given by figure 2.

Figure 2. Female Average life table 2010-12.

Life tables are generally constructed on five-age structure, for annuities calculation more accurate, we need to single years life table. Given that the question dealt in the present paper is annuities provisioning, only the curve beyond 60 years old are considered.

From five-years to single-years life table

The passage is from five years to single years tables can be done by various methods: Karup-king, Lagrange and Spargue. As we are dealing with pensioner population aged over 60, the Karup-King formula is the most appropriate method.

Table 1

Karup-King interpolation coefficients

	first group N0			Middle group Ni			Last group Nk		
	N0	N1	N2	Ni-1	Ni	Ni+1	Nk-2	Nk-1	Nk
1st fifth	0.344	-0.208	0.064	0.064	0.152	-0.016	-0.016	0.112	0.104
2nd fifth	0.248	-0.056	0.008	0.008	0.224	-0.032	-0.032	0.104	0.128
3rd fifth	0.176	0.048	-0.024	-0.024	0.248	-0.024	-0.024	0.048	0.176
4th fifth	0.128	0.104	-0.032	-0.032	0.224	0.008	0.008	-0.056	0.248
5th fifth	0.104	0.112	-0.016	-0.016	0.152	0.064	0.064	-0.208	0.344

Source : SHROCK and al. (1993)

The number of the deaths occurred in five-year's age group, can be interpolated by as following:

For medium age groups [60 – 80], we use the "Middle group" coefficients shown in table 1. For each five years age group [x, x+5[, the observed number of deaths $D(x,x+5)$ can be broken-out on 5 subsets corresponding for 5 single age groups. The number of death at each single age group d(i,i+1), i=x,x+1, …, x+4 can be deduced by using the following formula:

$$d(x, x + 1) = 0.064 * D(x - 5, x) + 0.152 * D(x, x + 5) - 0.016 * D(x + 5, x + 10)$$

$$d(x + 1, x + 2) = 0.008 * D(x - 5, x) + 0.224 * D(x, x + 5) - 0.032 * D(x + 5, x + 10)$$

$$d(x + 4, x + 5) = -0.016 * D(x - 5, x) + 0.152 * D(x, x + 5) - 0.064 * D(x + 5, x + 10)$$

To break out the death number for the extreme age group [80 − 85[, we use the "Last group" Coefficients. The formula will be:

$$d(x, x + 1) = -0.016 * D(x − 10, x − 5) + 0.112 * D(x − 5, x) + 0.104 * D(x, x + 5)$$
$$d(x + 1, x + 2) = -0.032 * D(x − 10, x − 5) + 0.104 * D(x − 5, x) + 0.128 * D(x, x + 5)$$
$$d(x + 4, x + 5) = 0.064 * D(x − 10, x − 5) − 0.208 * D(x − 5, x) + 0.344 * D(x, x + 5)$$

Once the detailed age deaths distribution is obtained, we recalculate the number of surviving (l_x) for detailed age x, by:

$$l_{x+1} = l_x − d(x, x + 1)$$

That allows deducing the detailed age death probability: $q_x = \frac{d(x,x+1)}{l_x}$ which is represented in figure 3.
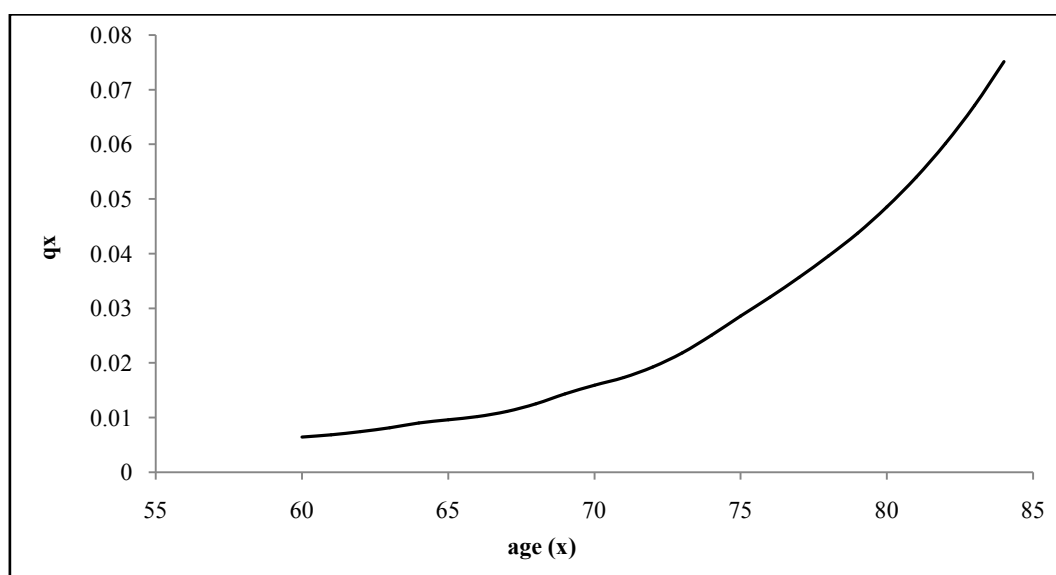

Figure 3. Single age life-tables for female population (60 − 85 years).

After getting a single age life- table, we will fit it using Makeham model (the model most used by actuaries). Recall that Makeham developed the Gompertz model, introducing an accident factor which is independent of age. Thereafter we will estimate Makeham model parameters.

Fitting life table with Makeham model

We remind that our final objective is not to construct a best life table, but just to show the impact of use of the continuous time calculation on life annuities calculation. So, we use simply the model the most commonly used for mortality fitting. This model is also adequate for Algerian data.

Makeham mortality model. The formula proposed by Makeham, were an extension for the Gompertz formula. Gompertz (1825) proposed that mortality rate is in exponential relation with age (x):

$$\mu_x = B\, c^x$$

After, Makeham (1867) modified this formula by introducing accident factor which is supposed to be independent from age.

$$\mu_x = A + B\, c^x$$

with B > 0, et C >1. 0<A<1

Fitting of the Average female life table 2010 – 2012. As mentioned by Coale & Guo (1989), Coale & Kikser (1990), the mortality trend changes starting from a specific age, generally between 80 or 85. Beyond this age, a specific mortality models are used (Quashie & Denuit, 2005). We aim to extrapolate the mortality curve until older ages with optimal quality. The idea is to fit the available series until the age of 80. To optimize the quality of the extrapolated data is preferable to use the slice [60, 80[to extrapolate the mortality until the older ages and use the slice [80, 85[to test the obtained result. Obtain a performance in the slice [80, 85[is supposed allow to get the same for the rest of the series.

To estimate model parameters we must write the formula in logarithmic form:

$$\ln(\mu_x - A) = \ln(B) + x.\ln(c)$$

Then, we have linear equation, the parameters *A, B* and *c* will be estimated by Least Square method. We have to minimize, for each age x, the difference between the original value given by the equation and the fitted line as shown in figure 4.
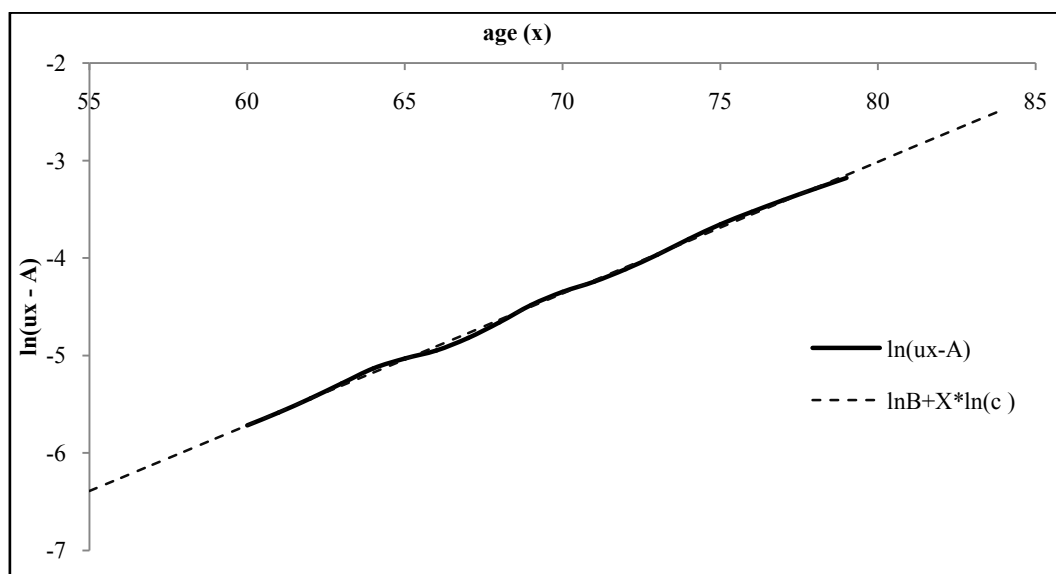

Figure 4. fitting by linearization of the mortality curve.

The parameters A, B and C are determined using the XL-Solver. The obtained results are shown in table 2. The corresponding curve is represented in figure 5.

Tabe 2

Makeham model parameters estimation

Makeham estimated parameters	
A	0.00315545
B	9.9067E-07
C	1.14469935

We observe that the fitted curve adheres perfectly the original curve, but after the age of 80, the curves diverge. That confirms that mortality trend changes behind this age, and other models may be used to complete the curve until older ages.
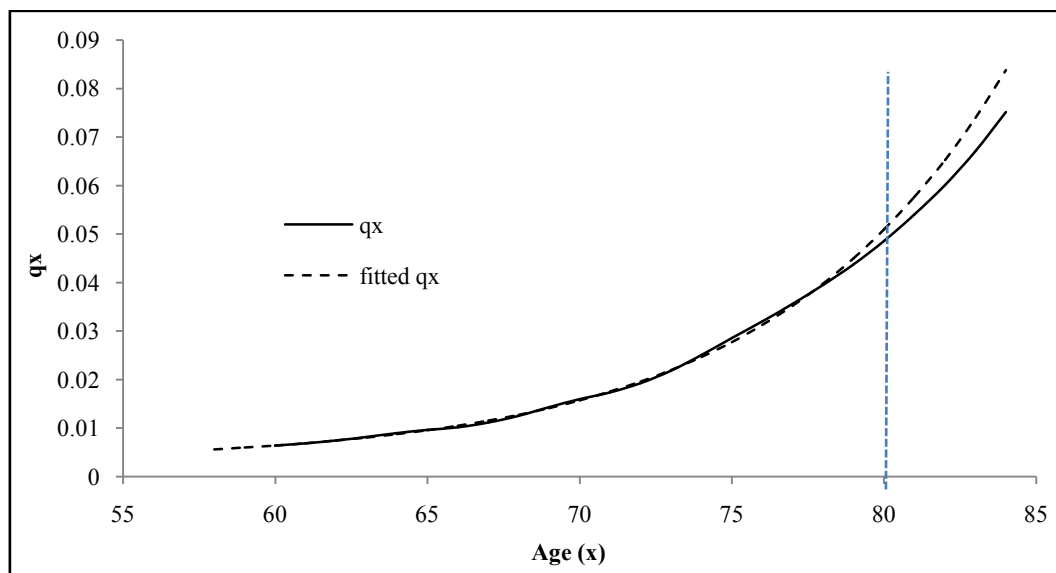
Figure 5. Fitting mortality curve using Makeham model.

Mortality extrapolating to the older ages

Counter to the fitting process, mortality extrapolating to the older ages is very sensitive to the choice of the extrapolating method. Here, we will present the methods the most commonly used.

Extrapolation Models. Various models were proposed to describe mortality evolution in the older ages. Quashie et Denuit (2005) made a large description for all models that can be used for. We remind that our objective is not to optimize the quality of the estimated mortality model, we reduce the set of the models to use for extrapolate mortality to the older ages. So, we have to compare between three models that we judged easy and adequate to the structure of our data: Kannisto model, Denuit & Goderniaux model and Coale & Kisker model.

• Kannisto model (1994)

The formula proposed by Kannisto (1994) is a simplified logistic formula. The instantaneous mortality rate is given by:

$$\mu_x = \frac{a.\,e^{bx}}{1 + a.\,(e^{bx} - 1)}$$

The logarithmic form :

$$ln\left(\frac{1}{\mu_x} - 1\right) = ln\left(\frac{1-a}{a}\right) - bx$$

Fitting the first terms of this equation linearly we obtain the formula parameters.

• Denuit & Goderniaux model (2005)

The proposed formula is based on polynomial description of the death rate logarithm:

$$\ln q_x = a + bx + cx^2$$

• Coale & Kisker Model (1990)

Coale & kisker based their formula on the mortality rate at age 79:

$$\hat{u}_x = \hat{u}_{x-1} \exp\big(k_{80} + s.\,(x - 80)\big) \; x = 80,81, \dots ,109.$$

K_{80} represents mortality growth rate at the age 80:

$$k_{80} = \frac{ln\left(\frac{\hat{u}_{80}}{\hat{u}_{65}}\right)}{15}$$

The authors fixed the mortality rate for an ultimate age of life:

$$\hat{u}_{110} = \begin{cases} 1.0 \ men \\ 0.8 \ women \end{cases}$$

For the present work, as it difficult to fix this parameter, it will be fixed automatically in the sense to optimize the quality of the estimated data, measured here by the Least Square for the slice [80, 85[.

S parameter is given by:

$$s = -\frac{ln\left(\frac{\hat{u}_{79}}{\hat{u}_{110}}\right) + 31 k_{80}}{465}$$

Extrapolation Result. The parameters of all these three models are estimated by optimization process. The objective is to minimize the sum of the absolute errors for the slice [65, 80[, that serve as parameters estimation base. However, for the Coale & Kisker model, the optimized function were the sum of absolute error for [80, 85[. These last sums were used as a common comparison base between the three models.

Tabe 3

Extrapolation models parameters estimation results

Kannisto		Denuit & Gourniaux		Coale & Kisker	
a	4.2023E-06	A	-12.08173408	k80	0.10914986
b	0.11809441	B	0.099357857	s	0.00109056
		C	0.118861882	u110	2.19115453
Sum_Abs_Error	0.0173	Sum_Abs_Error	0.0105	Sum_Abs_Error	0.0008

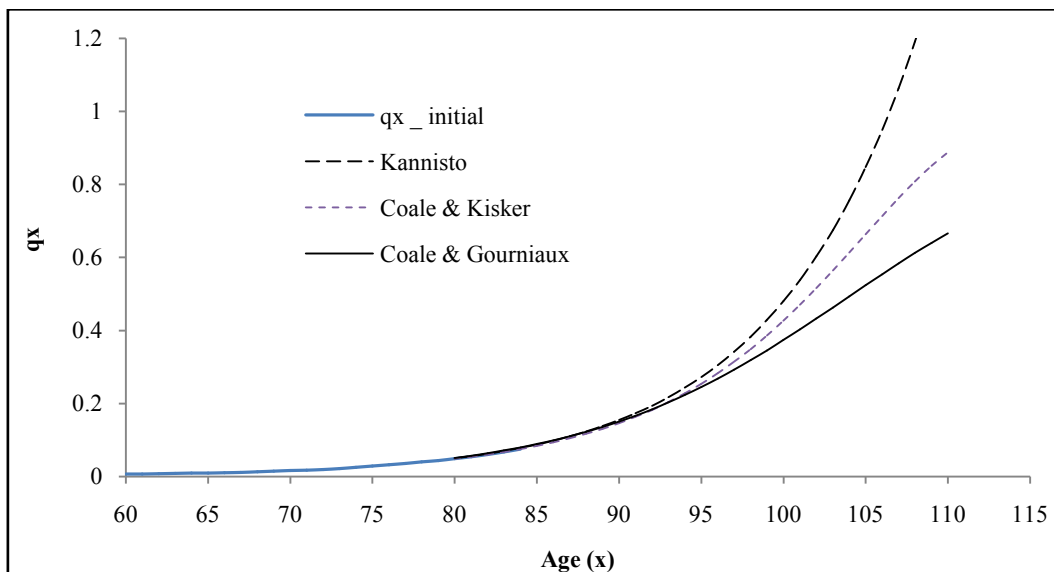We observe that the C&K model give the best result as shown in table 3. And figure 6.

Figure 6. Old ages mortality extending.

The three models give the approximately the same performance until the age of 93 years old. Behind this age, the curves diverge. C&G model seems give better old age mortality description comparatively with the rest of models. C&G model seems sub estimate mortality, counter to Kannisto model that give under estimate mortality rate at very older ages.

The result given by the C&K model is determined by the value supposed for μ_{110}. By the following graph, we show the sensibility of the older age mortality curve for this value.

We observe (see. Figure 7.) that for u_{110} supposed equal to 0.8, as done by Coale & kisker in their original paper, the C&K model give the same result like C&G. only, for 2.19 the model obtain his optimal result.

Until now, we have a single year complete life table. Using this table, we will estimate mathematical reserves for annuitants' portfolio that we present in the next point.
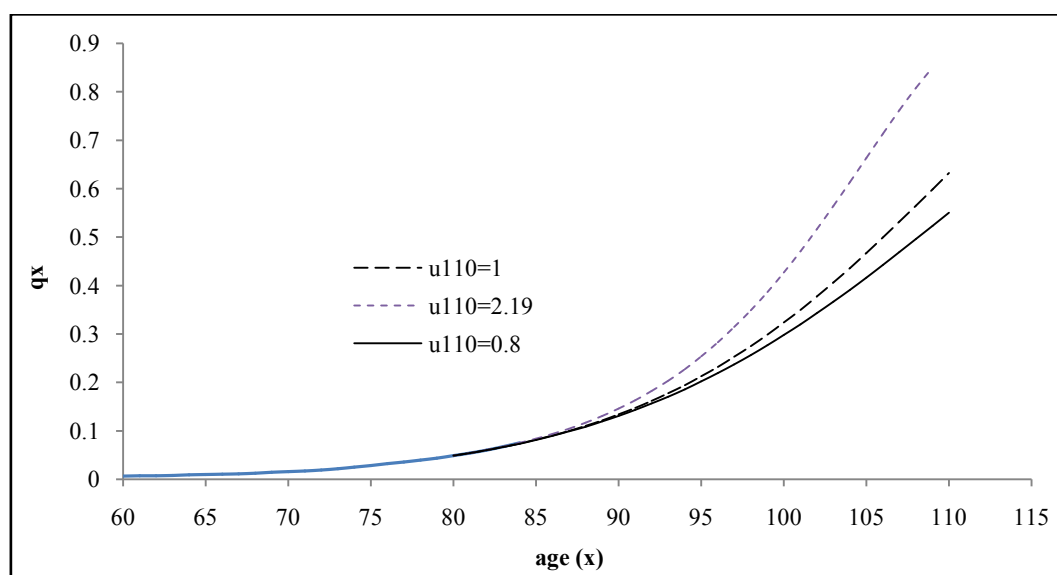
Figure 7. Coale & Kisker with different values of u110.

Annuitants' portfolio presentation

The portfolio that we use by the present paper is composed by 64 annuitants sampled from the portfolio in pay of an Algerian life insurance company. We have only withdrown those who receive more than 5000 dzd by quarter.

For technical considerations, this portfolio is supposed updated by 31.12.2012.

- Contract details and simplifying hypotheses:

By the present work, we suppose that all the annuitants have finished paying their subscriptions. That means that we are dealing with payment phase portfolio. Once retirement age reached, the annuitant receives his first annuity. After exactly 3 months, he receives the second and so on (quarterly annuity). In discrete time calculation for similar case, we use generally two approaches': annual calculation or quarterly calculation. Here, we use the most adequate to the payment time distribution. Therefore, in first, we must deduce the quarterly life table. This method is supposed to improve the calculation accuracy compared with the first method. If our objective is to demonstrate the utility of continuous time calculation on annuities provisioning, we must take for comparison the best result that we can get using discrete time approach. This method still based on restrictive hypotheses. It supposes that the individuals were born in the same quarter are exposed to the same

death risk. In others terms, the death risk is supposed constant during each quarter-age. By the same way, this calculation method is based on annuitants groups. Calculation for each group is done on average, i.e. amounts, date …etc. The continuous time calculation allows considering the exact date of each event. And the death risk at each point of the insured period.

The amount of the annuity is defined by the value of all the annuitant subscriptions during the payment phase capitalized until this 60th anniversary.

For the annual interest rate, we take 3%, for all operations.

Mathematical Reserves Calculation

Discrete time Approach

The principle of equity should be adopted in every aspect of managing a contract. So there must be a mathematical provision of each contract, the latter allows, on an actuarial basis and together with future premiums to fund future payments. The actuarial value is rationally calculated with prudent technical basis (Olivieri et Pitacco, 2008).

- Reminder accounting reserves method

In order to assess the liabilities, we will work with the prospective method which allows us to projected future so was able to assess future liabilities of the company to its customers, which is defined by the difference between future risks to cover (annuity payment) and future commitments of the insured (premiums to pay).

Using international actuarial notations, the formula for the prospective mathematical reserve is written as follows:

$$_tV_x = \ddot{a}_{x+t]} * R - P\,\ddot{a}_{x+t]}$$

$_tV_x$: Mathematical reserves calculated yet t.

$\ddot{a}_{x+t]}$: Expected present value of an annual or quarterly annuity of 1 DZD at time t + x.

R: The rent paid by the insurer.

P : The annual premium paid by the insured.

In our work, we are interested in the age group 60 and over, thus the phase of portfolio construction is completed. So that policyholders have completed their commitments (payment of premiums). Therefore, the mathematical reserve is equal to the expected present value of the remaining terms to pay the rent.

We obtain;

$$_tV_x = \ddot{a}_{x+t]} * R$$

By using commutations, the formula of annuity and using appropriate commutations is obtained;

$$l_x\ddot{a}_x = (l_x v^0 * R + l_{x+1} * v^1 * R + l_{x+2} * v^2 * R + \cdots + l_\omega * v^{\omega-x}) * R$$

ω being the age limit of the table;

We suppose that the annuity (R) is equal to 1 dzd.

We remind that v^n represent the actualization factor;

$$v^n = \frac{1}{(1+i)^n} = (1+i)^{-n}$$

We get:

$$l_x\ddot{a}_x = l_x v^0 * l_{x+1} * v^1 + l_{x+2} * v^2 + \cdots + l_\omega * v^{\omega-x}$$

$$\ddot{a}_x = 1 + \frac{l_{x+1} * v^1 + l_{x+2} * v^2 + \cdots + l_\omega * v^{\omega-x}}{l_x}$$

$$\ddot{a}_x = 1 + \frac{l_{x+1} * v^1 + l_{x+2} * v^2 + \cdots + l_\omega * v^{\omega-x}}{l_x} * \frac{v^x}{v^x}$$

By using the following commutations:

$$D_x = l_x * v^x$$

D_x Measures the intensity of the financial and lifetime risks for an individual at age x.

We have also :

$$N_x = \sum_{k=1}^{\omega-x} D_{x+k}$$

We get ;

$$\ddot{a}_x = \frac{N_x}{D_x} . R$$

R : Annual annuity amount.

These formulas are kept for quarterly time accounting, only time and age indicators must be expressed on quarters. It is the same for life table. For this, we keep the hypothesis of risk death constancy during each year.

In table 4., we show calculation process and results obtained for an annuitant aged 280 quarters (70 years) :

Tabe 4

Simulation of life annuity contract – discrete time reserving

				i =	0.030			
				i*=	0.007			
				V=	0.993			
x	Lx	dj	Vj	Dj	Nj	Nj/Dj	*Quarterely annuity*	MR (i)
280	90818.82	364.54	0.12630	11470.18	557961.97	48.64	5,160.50 DZD	251,030.37 DZD

To obtain the value of this contract by 31/12/2012, we have just to multiply the value of Nj/Dj by the quarterly annuity amount. The global value of the portfolio mathematical reserves is equal to:

$$MR = \sum_{i=1}^{64} MR_i$$

The global amount of the mathematical reserves for the considered portfolio is evaluated for 45.7 million DZD.

Continuous-time Approach

Finished with a continuous modeling of the provision, to do this we will estimate the instantaneous rate of death and assumed constant technical rate in continuous time. Then apply these results on our annuitant's portfolio and compare the results with those obtained with discrete time method.

To calculate the continuous mathematical reserves, we must use the instantaneous mortality rate (μ_x) that we got by life table fitting with Makeham formula and extrapolated to the older ages by Coale & Kisker model.

So, the calculation will be done in two steps. In first, we must express the expected present value of the pensions paid after the age of 80 in continuous form (Coale & Kikser formula). Then, the obtained equation will be integrated on the formula based on Makeham's model.

- Continuous technical rate

Since the annuity is considered continuous, we will use instant technical interest rate, and for this we must spend capitalization rate discrete capitalization rate continuously, knowing that the capitalization is compound interest.

We make;

C_o: Initial Capital at time 0,

C_t: Capital at time t, (t > 0),

i: interest rate,

$$C_o = C_t \ (1 + i)^t = C_0 e^{rt}$$

If we know that:

$$r = \ln(1 + i) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{instantaneous rate.}$$

The capital at time *t* can be written like a following differential equation

$$d \ C_t = r \ C_0 \ dt$$

We obtain the following equation:

$$C_t = C_0 \ e^{(\int_0^t r(s) ds)}$$

So, the present value is given by:

$$C_0 = \frac{C_t}{e^{(\int_0^t r(s) ds)}} = C_t \ e^{-(\int_0^t r(s) ds)}$$

Continuous mortality rate

The instantaneous rate of death allows us to estimate the mortality variation between x and (x + t).

We have:

$$\int_0^t \mu_{x+s} \ ds$$

We replace μ_{x+s} by the value given by makeham model.

$$= \int_0^t (A + Bc^{x+s}) ds$$

$$= At + B \int_0^t \exp((x + s) \ln c) \ ds$$

$$= At + \frac{B}{\ln c} \left\{ e^{(x+t)\ln c} - e^{x \ln c} \right\}$$

$$= At + \frac{\alpha}{\ln c} \left\{ c^{x+t} - c^x \right\}$$

We obtain (Hess, 2000):

$$\int_0^t \mu_{x+s} \ ds = At + \frac{B}{\ln c} \ c^x \ (c^t - 1)$$

Usually, annuity pricing formula is written as following (Justens et Hulin, 2003):

$$\bar{a}_x = \int_0^\infty e^{\int_0^t -(\mu_{x+\tau}+r(\tau))d\tau}\, dt$$

with

$r(\tau)$ represents the instantaneous technical interest rate.

$(\mu_{x+\tau})$ Represents the instantaneous mortality rate.

But, by the present work, the instantaneous mortality rate is defined by two equations:

For x : [60, 80[: $\mu_x = 0.00315 + 9.906(e-07)\,1.1445^x$ (Makeham)

Beyond 80 years:

$$\mu_x = \mu_{x-1}\exp\big(0.109 + 0.00109.(x-80)\big) \text{ (Coale \& Kisker)}$$

To facilitate the calculation process, we propose a fitting model to pass from two models to one. Makeham formula is again used for modeling mortality evolution for age slice 60 and over. The fitting results are shown in figure 8.
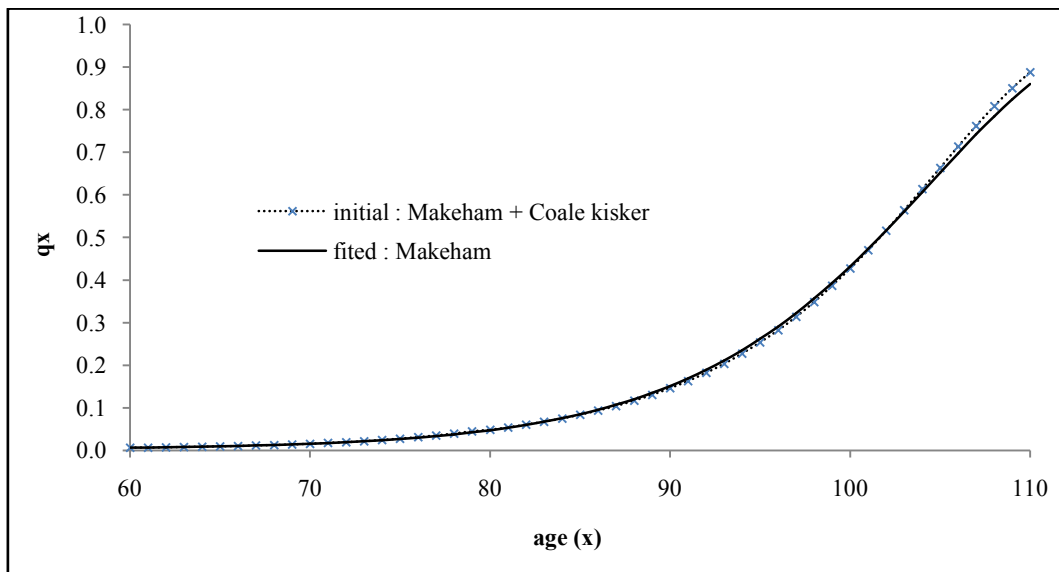

Figure 8. Fitting mortality for age range [60, 110] by one Makeham model.

We observe (table 5.) the goodness of the adjustment; the average error is less than 0.0012 before the age of 105, after this difference increases slightly to be close to 0.02. This difference in older ages can be neglected.

The parameters of the new model are given by:

Tabe 5

Makeham model parameters estimation for 60 – 110 age range

New Model Parameters	
A'	0.002623604
B'	2.07137E-06
C'	1.133295399

This new equation allows writing:

$$\overline{a_x} = \int_{x-60}^{\omega-(x-60)} e^{\int_0^t -(\mu_{x+\tau}+r(\tau))d\tau}\, dt$$

Mathematical reserves expressed in continuous time can be written:

$$\overline{a_x} = \left[\int_{x-60}^{\omega-(x-60)} e^{\int_0^t -(\mu_{x+\tau}+r(\tau))d\tau} dt\right] * R_T$$

R_T : Quarterly annuity amount.

If we suppose that instantaneous interest rate $(r = \ln(1 + i))$ is constant.

$$\overline{a_x} = \int_{x-60}^{\omega-(x-60)} e^{\left[-At-\frac{Bc^x}{lnc}e^{tlnc}-\frac{r(t)^2}{2}\right]}dt$$

The finale formula :

$$\overline{a_x} = \left[e^{\left[-At-\frac{Bc^x}{lnc}e^{tlnc}-\frac{r(t)^2}{2}\right]}\right]_{x-60}^{\omega-(x-60)}$$

The mathematical reserve in continuous time should be estimated between two time points corresponding to two age point x and x+t by using the continuous interest rate. We get :

$$\bar{a}_x = \left[e^{\left[-At-\frac{\propto c^x}{lnc}e^{tlnc}-\frac{r(t)^2}{2}\right]}\right]_{x-60}^{(x+t)-60}$$

x : is the calculation date.

$x + t$: First payment date.

Finally, the value of mathematical reserve for each annuitant aged x, is given by the multiplication of a_x by the quarterly annuity:

$$MR_i = \left[e^{\left[-At-\frac{Bc^x}{lnc}e^{tlnc}-\frac{r(t)^2}{2}\right]}\right]_{(x+t-60)}^{(x-60)} * R_i$$

The result obtained with the same annuitant that we dealt in the case of discrete time calculation is given by table 4.

Table 4

Simulation of life annuity contract – continuous time reserving

Birth date	AGE by 31/12/2012	First annuity date	Quarteral incomes	Mathematical Reserves (Discrete time)	Mathematical Reserves (Continuous time)
01/02/1942	70	30/01/2002	5,160.50	251,030.37 (DZD)	236,482.98 (DZD)

We observe that the mathematical reserve for an annuitant aged 70 years have decreased by more than 5% when we passed on continuous time calculation. The amount for the global population has passed from 251.000 to 236.000 DZD.

Conclusion

All institutions engaged in economic activity are confronted to the risk of loss. To escape to this situation all risk sources might be evaluated. Insurance companies are not immune to this issue, as we explained in our work. Life annuities, like all life insurance long-term contracts, need an especial consideration in pricing and funding evaluation.

The set of life insurance companies in Algeria have a various options to improve their service and the

security of their activities. If the financial aspect is not able to bring a great improvement, the demographic aspect may have an important role on calculation accuracy. We recall that annuities calculations in Algeria still based on old life table (TV- 1997-1999) constructed by the National Council for the Insurance. By the present work, we presented a new life table that can serves in annuities calculation. The Algerian insurance regulation imposes to all insurers the use of this life table. By there is other solution to improve pricing and funding process using the same life table, just by passing to continuous time calculation methods.

By the present work, we showed that continuous time calculation allows getting reduced mathematical reserves compared with discrete time calculation by about 5% for the considered portfolio. This result of 5% were got compared with the best that we can do with discrete time calculation. If we consider the effort to make to pass to quarterly life table, the advantages of time continuous table are multiplied. The complication increase when we deal with monthly annuities and with different payment dates. With time continuous calculation, life insurance companies can better estimate their future engagements, and propose reduced annuity prices using the same life table, i.e. regulatory table. The most important point is to can release additional reserves without affecting his financial equilibrium.

We might say that the estimation of the mathematical reserve continuously gives us more affine results compared with discrete time calculation. For each contract, this method allows to estimate exactly how much to satisfy the future payments. Therefore, it allows more equitable pricing, and more flexible contract management. In terms of pricing, the reduction the use of continuous calculation method that allows reduced prices might increase the attractiveness of the proposed products especially in the Algerian market. The number of life insurance companies is increasing, and it is only by the similar practice that a company can take a lead.

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