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An Improved Technique of Calculating Deflections of Flexural Reinforced Concrete Elements Made of Conventional and High-Strength Concrete

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Abstract: The paper presents an improved technique of calculating total deflections of flexural reinforced concrete elements that takes discrete crack formation into account. The technique is based on determining the curvature of the cross section of reinforced concrete elements with cracks and fissures in the area between cracks. The curvature of the element is calculated using a non-linear function of the deformation of concrete under compression. Approximating dependency of concrete resistance on compression developed by one of the authors is presented. An algorithm of finding the curvature and formulas for calculating curvature and deflection are provided. The function of the curvature distribution along the length of a flexible element is proposed by the authors. The paper also presents the results of the author's experimental research. The characteristics of samples tested are described. The experimental research results of deflections of flexural reinforced concrete elements made of conventional and high-strength concretes are presented. Comparison of the values calculated using the technique with those obtained from the experimental research as well as those calculated according to existing regulations in Russia, USA and Europe is drawn.

Key words: Deflection, concrete tension diagram, flexural elements.

1. Introduction

The construction of modern buildings that require high reliability and responsibility, such as tall, entertainment, or Olympic sports facilities, assumes using the most modern building materials, construction technologies and methodologies as well as building structures' operation assessment techniques. Ultra high-strength concrete (HPC—high performance concrete) with improved physical and chemical properties is used along with HSCs (high-strength concretes) and ordinary NSCs (normal strength concretes) are used in such structures in the global construction industry.

The use of high-strength concrete allows reducing the cross-sectional designs, which leads to impaired stiffness characteristics. As a result, deflections in some cases become the determining factor of geometric forms and reinforcement. Unfortunately, the techniques of calculating high-strength concrete structural rigidity parameters are reflected extremely controversially in the normative literature and are simply not available in some countries.

The techniques of calculating deflection of reinforced concrete flexural elements are inextricably linked with determining the real tension-deformed state in any section of the element under consideration along the entire structures, cracks, as well as the physical characteristics of reinforced concrete. Thus, the accuracy and convergence of research results depends directly on the introduction of realistic concrete and its reinforcement deformation models into these techniques.

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2. Description of the Improved Technique of Calculating Deflections of Flexural Reinforced Concrete Elements

There are many different techniques to determine the dependency of the deformation of compressed and strain concrete and reinforcement as described in domestic [1-4] and foreign [5-17] research papers and legal references. The following approximating concrete deformation as a function of its compression was proposed in the SGASU (Samara State University of Architecture and Civil Engineering):

$$\sigma_b(\varepsilon) = a \cdot \varepsilon^b \cdot \exp\left(\frac{-b \cdot \varepsilon}{p}\right)$$

where, a, b and p are determined from the calculation of the prerequisites set forth in the normative materials, $\sigma_b(\varepsilon)$—tension in the concrete dependent on the relative deformation of ε.

This function accords fairly well with experimental data as well as proposals in Refs. [4, 11, 18-22] and is a lot easier for use in practical calculations due to computers.

The diagram shown in Fig. 1 based on the function being proposed and refined using data in Refs. [23-25] is taken as the basis of concrete deformation diagram.

Based on the deformation diagram proposed in SGASU, the technique of calculating bending deflections of reinforced concrete structures was improved.

The following assumptions were made:

(1) A concrete bendable freely supported beam with two loads equidistant from the poles is being considered;

(2) There are two characteristic areas along the beam: area I with no cracks in shear force zone and area II with cracks in pure bending zone;

(3) Linear curvature distribution in area I is assumed;

(4) Sinusoidal curvature distribution in area II is assumed;

(5) Characteristics of a sinusoidal law are determined based on calculating the height of the whole compressed concrete area with the cracks and of a compressed concrete area between them;

(6) Curvature R (in cross section of an area with a crack) is determined based on the law of equal forces generated by compressed concrete and pre-stressed reinforcement;

(7) Curvature R_1 (in cross section in the middle of an area between cracks) is determined from the condition that the values of relative deformations of strain concrete do not exceed their limits (with $\varepsilon_{bt} = \varepsilon_{bt2}$), and the hypothesis of plane sections applies only to compressed concrete.

The algorithm for determining bending deflections of reinforced concrete structures is implemented in MathCAD 14 software package.

The equations of equilibrium in cross section of an area without cracks take from Eq. (1).

$$\begin{cases} \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx - \int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy - f_s\left[\frac{\varepsilon_{bt}}{t}\cdot(h_0-k)\right]\cdot A_s + \\ + f_{sc}\left[\frac{\varepsilon_b}{k}\cdot(k-a_2)\right]\cdot A_{sc} = 0; \\ \int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot b\cdot dx \cdot \left[\left(\frac{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot x\cdot dx}{\int_0^k \sigma_b\left(\frac{\varepsilon_b}{k}\cdot x\right)\cdot dx}\right) + h_0 - k\right] + \\ + f_s\left[\frac{\varepsilon_b}{k}\cdot(k-a_2)\right]\cdot A_{sc}\cdot(h_0-a_1-a_2) - \\ - \int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot b\cdot dy\cdot\left(h0 - k - \frac{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot y\cdot dy}{\int_0^t \sigma_{bt}\left(\frac{\varepsilon_{bt}}{t}\cdot y\right)\cdot dy}\right) - M_u = 0 \end{cases} \quad (1)$$

In Eq. (1), $\sigma_b(\varepsilon_{bx})$—tension in compressed concrete, $\sigma_{bt}(\varepsilon_{bty})$—tension in strained concrete, $f_s(\varepsilon_s)$ —tension in strained reinforcement, $f_{sc}(\varepsilon_{sc})$ —tension in compressed reinforcement, $\varepsilon_{bx} = \frac{\varepsilon_b}{k}\cdot x$ —current value of relative deformation in compressed concrete, $\varepsilon_{by} = \frac{\varepsilon_{bt}}{t}\cdot y$ —current value of relative deformation in strained concrete, $\varepsilon_{sc} = \frac{\varepsilon_b}{k}\cdot(k-a_2)$ —current value of relative deformation in the compressed reinforcement,

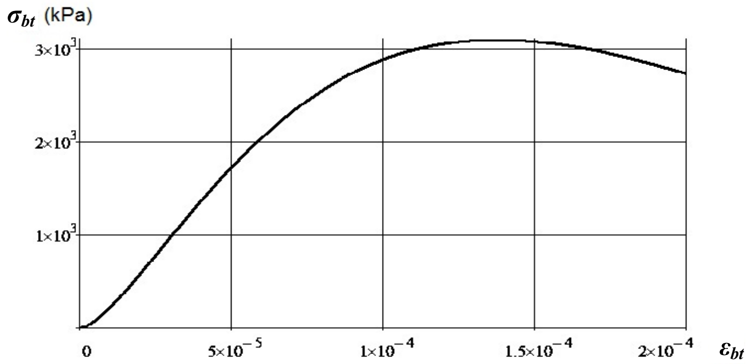

Fig. 1 B-90 class concrete stretch deformation diagram based on the diagram proposed in Ref. [18] and using studies in Refs. [23-25].

$\varepsilon_s = \frac{\varepsilon_{bt}}{t} \cdot (h_0 - k)$—current value of relative deformation in strained reinforcement, ε_{b2}—limiting value of relative deformation of compressed concrete, ε_{bt2}—limiting value of relative deformation of strained concrete, x—current height of the compressed zone of concrete, y—current height of the strained area of concrete, k—height of the compressed area of concrete, t—height of the strained area of concrete, h—full height of cross section, b—width of cross section, a_1 and a_2—thickness of the protective concrete layer of strained and compressed reinforcement respectively, A_s and A_{sc}—area of strained and compressed reinforcement respectively.

Solution of equations set Eq. (1) gives all the values k and ε_b taking into account nonlinear deformation of concrete and reinforcement upon compression and stretching using the corresponding deformation diagrams. From these values, the authors can find curvature R using Eq. (2):

$$R = \frac{\varepsilon_b}{k} \tag{2}$$

Proceeding from these boundary conditions, it is assumed that there will be no new cracks in an area between cracks. The area size equals distance $l_s(\varepsilon_b)$ between cracks. The quantity of the areas is determined using Eq. (3):

$$aa = Floor\left(\frac{L - 2m}{l_s(\varepsilon_b)}, 2\right) \tag{3}$$

The tension-deformed condition in the middle of an area between cracks is described by Eq. (4):

$$\int_0^{k1} \sigma_b\left(\frac{\varepsilon_{b1}}{k_1} \cdot x\right) \cdot b \cdot dx - \int_0^{H-k1} \sigma_{bt}\left(\frac{\varepsilon_{bt2}}{H - k_1} \cdot y\right) \cdot b \cdot dy - f_s\left[\frac{\varepsilon_{b1}}{k_1} \cdot (h_0 - k_1)\right] \cdot A_s + f_{sc}\left[\frac{\varepsilon_{b1}}{k_1} \cdot (k_1 - a_2)\right] \cdot A_{sc} = 0 \tag{4}$$

Let the authors describe the pure bending zone using the following boundary conditions m_1 and m_2 defined under Eqs. (5) and (6):

$$m_1 = \frac{1}{2} \cdot (L - aa \cdot L_s) \tag{5}$$

$$m_2 = aa \cdot L_s + \frac{1}{2} \cdot (L - aa \cdot L_s) \tag{6}$$

where, m_1, m_2—distances between the left and right pole and the nearest crack respectively, L—rated span of the beam.

The function of curvature changes in the pure bending zone is assumed to have sinusoidal shape as per Eq. (7):

$$P(x) = \frac{R - R_1}{2} \cdot \sin\left(\frac{-2\pi}{L_s} \cdot x + \frac{\pi}{2} + \frac{2 \cdot \pi \cdot m}{L_s}\right) + \frac{R_1 + R}{2} \tag{7}$$

where, R is curvature in an area with a crack, R_1 is curvature in the middle of an area between cracks.

The function of curvature changes in an area without cracks is assumed to look like:

$$P_1(v) = \frac{R}{m_1} \cdot v \tag{8}$$

$$P_2(v) = \frac{R \cdot (v - L + m_1)}{m_1} + R \tag{9}$$

where, P_1 (v), P_2 (v)—functions of curvature changes in the left and right areas of the beam without cracks, respectively.

Total dependency of curvature distribution $W(v)$ along the beam (Fig. 2) is obtained using Eq. (10):

$$W(v)=\begin{cases} P_1(v) & if\ \ 0\leq v\leq m+\frac{1}{2}\cdot(L-2m-aa\cdot L_s) \\ P(v) & if\ \ m+\frac{1}{2}\cdot(L-2m-aa\cdot L_s)\leq v\leq 4L_s+m+\frac{1}{2}(L-2m-aa\cdot L_s) \\ P_2(v) & if\ \ 4L_s+m+\frac{1}{2}\cdot(L-2m-aa\cdot L_s)\leq v\leq L \end{cases} \quad (10)$$

To calculate total deflection of the structure using general rules of construction mechanics, the epures of changes in moment M_1 and lateral force Q_1 from an individual load as well as those in moment M_x and lateral force Q_x from external load are found.

The value of total deflection includes deflections resulting from the influence of bending moment FM_{max} and lateral force FQ_{max}.

Total deflection of the structure is calculated using Eq. (11):

$$F = FM_{\max} + FQ_{\max} \quad (11)$$

According to SP 52-101-2003 [4], deflection FM_{max} caused by bending deformation is found using Eq. (12) and deflection FQ_{max} caused by shear deformation is found using Eq. (13):

$$FM_{\max}=\begin{cases} \int_0^L W(v)\cdot M_1(v)dv, & M_u \geq M_{crc}(\varepsilon_{br}) \\ \int_0^L R_d\cdot M_1(v)dv, & M_u < M_{crc}(\varepsilon_{br}) \end{cases} \quad (12)$$

$$FQ_{\max}=\begin{cases} \int_0^L \frac{9\cdot I_{red}}{b\cdot H}\cdot\frac{Q_x(v)\cdot Q_1(v)}{M_x(v)}W(v)dv, & M_u \geq M_{crc}(\varepsilon_{br}) \\ \int_0^L \frac{3}{E_b\cdot b\cdot H}\cdot Q_x(v)\cdot Q_1(v)dv, & M_u < M_{crc}(\varepsilon_{br}) \end{cases} \quad (13)$$

where, I_{red}—normalized (reduced) moment of inertia of cross section, E_b—concrete elasticity module, $M_{crc}(\varepsilon_{br})$—value of the bending moment required for crack formation.

3. Results

3.1 Comparison of Results with Existing Calculation Technique

The results of the authors, investigation and the results of those of other authors were analyzed in order to confirm that the proposed technique of calculating deflections of flexural reinforced concrete structures is legitimate for application. Initial characteristics of samples investigated at SGASU are given in Tables 1-3.

Physical characteristics of the sample materials at SGASU are given in Tables 4 and 5.

3.2 The Proposed Technique Developed in SGASU

Results of the analysis of experimental data of various authors using existing normative documents and the proposed technique developed by authors in SGASU are shown in Table 6.

4. Conclusions

The techniques stipulated by current normative documents considered in Table 3 have the results overestimated (up to 40% for plane) NSC series and unstable (overestimated to 35% or underestimated to 25%) results for HSC series.

For all experimental studies considered above, the technique proposed in this article provides considerably better convergence between the theory and experiment in comparison with other techniques.

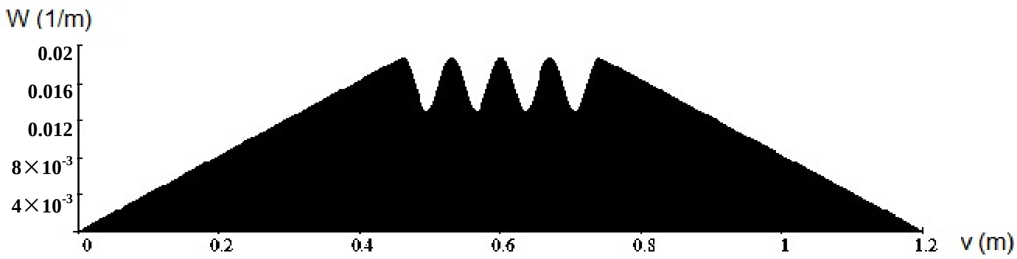

Fig. 2 Generalized epure of curvature changes along an element.

Table 1 Geometrical characteristics.

Mark of the sample	Span (m)	Section height (m)	Section width (m)
Об 1.1-1.3	1.2	0.22	0.12
Об 2.1-2.3	1.2	0.22	0.12
Об 3.1-3.3	1.2	0.22	0.12
Об 4.1-4.3	1.2	0.22	0.12
Об 5.1-5.3	1.2	0.22	0.12
Об 6.1-6.3	1.2	0.12	0.22

Table 2 Reinforcement.

Mark of the sample	Strain reinforcement	Compressed reinforcement	Lateral reinforcement
Об 1.1-1.3	1Ø12	1Ø10	Ø10s90
Об 2.1-2.3	1Ø16	1Ø10	Ø10s90
Об 3.1-3.3	1Ø20	1Ø10	Ø10s90
Об 4.1-4.3	2Ø12	2Ø10	Ø10s90
Об 5.1-5.3	2Ø16	2Ø10	Ø10s90
Об 6.1-6.3	2Ø25	-	Ø10s50

Table 3 Reinforcement percentage.

Mark of the sample	Strain reinforcement (%)	Compressed reinforcement (%)
Об 1.1-1.3	0.428	0.296
Об 2.1-2.3	0.761	0.296
Об 3.1-3.3	1.189	0.296
Об 4.1-4.3	0.85	0.592
Об 5.1-5.3	1.522	0.592
Об 6.1-6.3	3.71	-

Table 4 Concrete characteristics.

Mark of the sample	Strength compression of concrete (MPa)	Tensile strength of concrete (MPa)	Limiting relative compressive deformations of concrete	Limiting relative tensile deformations of concrete
Об 1.1-1.3	18.49	1.6	0.0033	1.24e-4
Об 2.1-2.3	18.49	1.6	0.0033	1.24e-4
Об 3.1-3.3	21.74	1.8	0.0034	1.26e-4
Об 4.1-4.3	64.1	2.85	0.0028	2.0e-4
Об 5.1-5.3	64.1	2.85	0.0028	2.0e-4
Об 6.1-6.3	64.1	2.85	0.0028	2.0e-4

Table 5 Reinforcement characteristics.

Mark of the sample	Area of longitudinal working reinforcement (mm^2)	Elasticity limit (MPa)	Rupture strength (MPa)	Relative deformations of reinforcement at fluidity	Relative deformations of reinforcement at rupture
Об 1.1-1.3	113.04	445.4	649.33	2.639e-3	0.038
Об 2.1-2.3	200.96	348.4	631.97	1.971e-3	0.078
Об 3.1-3.3	314.0	477.0	646.5	2.371e-3	0.069
Об 4.1-4.3	226.08	445.4	649.33	2.639e-3	0.038
Об 5.1-5.3	401.92	442.5	631.97	2.479e-3	0.078
Об 6.1-6.3	981.20	387.3	666.5	2.31e-3	0.108

Table 6 Results of experimental research of deflections.

Experimental studies	Marks of series	Deviation of deflection as per specified method from experimental value (%)				
		Normative document				Proposed method [24]
		СП 52-101-2003 [4]	СНиП 2.03.01-84 [3]	ACI 435R-95 [7, 8]	Eurocode 2 [11]	
SGASU [25-27]	Об 1.1-1.3	40.84	25.08	-15.47	-6.90	-4.72
	Об 2.1-2.3	35.53	15.52	-19.52	-9.75	-5.47
	Об 3.1-3.3	30.81	18.86	-28.49	-14.29	-6.97
	Об 4.1-4.3	20.98	8.82	-19.46	-12.91	-5.62
	Об 5.1-5.3	23.91	17.46	-21.69	-8.52	-6.19
	Об 6.1-6.3	-0.81	-15.83	-33.40	1.32	-2.87
	ОБ 1.1, 1.2	44.42	8.58	-15.17	3.82	3.48
	ОБ 2.1, 2.2.	-1.41	-14.23	-36.90	-14.21	2.79
	Б1-Бп	-15.44	-24.92	-45.60	-34.66	5.31
	Б2-Бп	-4.00	-16.38	-44.71	-27.44	-2.85
	Б3-Бп	-12.16	-16.24	-47.35	-36.86	-8.99
Bondarenko et al. [28]	Б-1-1	41.53	9.05	-15.45	-10.67	9.70
Davydov and Donchenko [29]	И-1-0	-18.97	-34.70	-48.11	-36.85	9.47
Valovoy and Gerb [30-32]	БК	31.91	18.73	-15.12	-4.54	7.38
Vanus [33]	Б-I-1	43.94	1.98	-	-1.33	-5.76
	Б-II-1	4.39	-12.10	-	-15.03	-3.38
Ahmad and Barker [34]	LJ-6-16	37.01	-13.76	-13.05	-8.10	5.91
	LJ-7-31	32.85	-12.12	-15.99	-4.20	-10.19
	LJ-8-44	21.81	-12.28	-18.73	-2.53	-2.71
Ahmad and Batts [35]	LR5-19	28.58	-26.34	-22.24	-24.02	-4.56
	LR8-22	32.15	-22.27	-21.35	-19.41	-4.62
	LR11-24	34.87	-15.82	-19.06	-13.15	-1.69
Rashid and Mansur [36]	C211	3.94	-8.91	-31.58	-14.18	2.33
	C311	-11.71	-18.11	-38.68	-19.89	-8.83
	C411	-4.07	-9.48	-33.74	-12.54	-4.85
	C511	-13.53	-14.95	-38.55	-16.44	-5.12
	D211	11.64	-4.05	-28.18	-7.51	3.48
	E211	12.26	-5.70	-29.34	-9.91	2.33
Sato et al. [37]	V-01-10WB	-	13.19	13.35	17.97	7.95
	V-01-10DB	-	-2.14	-1.80	2.68	-7.23
	V-01-13WB	-	6.55	-5.43	8.98	-4.34
	V-01-16WB	-	6.50	-10.51	7.90	-6.45
	V-01-16DB	-	1.87	-13.93	4.38	-5.24

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