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Determination of Ash Contents in Coal by Means of Ordinary Kriging Method

Tomasz Niedoba and Tadeusz Tumidajski

Department of Environmental Engineering and Mineral Processing, Faculty of Mining and Geoengineering, AGH University of Science and Technology, Kraków 30-059, Poland

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Abstract: The ash contents in coal particles were examined in the paper dependably on particle size and its density. So, the two-dimensional regressive function $Z = Z(P, D)$ was the searched object, where Z is random variable describing ash contents, P—density and D—particle diameter. This dependence was determined based on experimental data concerning the coal of type 31. For this coal, the method of ordinary kriging was applied to calculate the values of random variable Z. This method required the proper selection of so-called variogram function, in which four forms were considered in this paper in purpose to select the best solution. The given results were then evaluated by the mean standard error value and compared with empirical data.

Key words: Ordinary kriging, coal, ash contents, geostatistics.

1. Introduction

In mineral processing, the various characteristics of particles decide about their separation. The research over the dependencies between ash contents in coal particles and their size and density will be the issue of the following paper. Let introduce the following random variables: D—size (diameter) of particle; P—density of particle; Z—ash contents in particle. On the basis of experimental research based on the material separation during which the ash contents for various particle sizes and densities were measured, the function $Z = L(P, D)$ will be determined. Theoretically this function may be given by several methods, like:

(a) By using of classical theory of two-dimensional regression, so searching for the function, i.e. linear one $z = ad + b\rho + c$, exponential one $z = ad^b\rho^c$ or others on the basis of the empirical data, where d is particle size, ρ—particle density, z—ash contents;

(b) By construction of three-dimensional distribution function $X = (D, P, Z)$, determining the function $L(D, P)$ on the basis of conditional regression, so

$$L(d,\rho) = \int_A \frac{zf(d,\rho,z)}{g(d,\rho)}dz \qquad (1)$$

where, A is set of values of random variable Z, $f(d, \rho, z)$ is density function of random variable X, $g(d, \rho)$ is density function of two-dimensional random variable $Y = (D, P)$.

For approximation of the density function of researched multidimensional distribution functions of random variables X and Y, the Morgenstern distribution functions can be applied [1-3], in which it is assumed that the density function of random variable Y is given by Eq. (2).

$$g(d,\rho) = f_1(d)f_2(\rho)[1+\alpha(1-2F_1(d)(1-2F_2(\rho)))] \qquad (2)$$

where, $f_1(d)$ is density function of random variable D, $f_2(\rho)$ is density function of random variable P and $F_1(d)$ and $F_2(\rho)$ are their distribution functions; $\alpha\in[-1, 1]$ is properly selected coefficient.

In the same way, the density function of three-dimensional random variable X is determined and it can be obtained that

Corresponding author: Tomasz Niedoba, Ph.D., main research fields: mineral processing, statistical and mathematical modeling, metrology, monitoring of pollution. E-mail: tniedoba@agh.edu.pl.

$$f(d,\rho,z)=\begin{bmatrix}1+\alpha_1(1-2F_1(d))(1-2F_2(\rho))+\alpha_2(1-2F_1(d))(1-2F_3(z))+\\+\alpha_3(1-2F_2(\rho))(1-2F_3(z))+\alpha_4\begin{pmatrix}1-2F_1(d)\\(1-2F_2(\rho))(1-2F_3(z))\end{pmatrix}\end{bmatrix}\cdot f_1(d)\cdot f_2(\rho)\cdot f_3(z) \quad (3)$$

where, $f_3(z)$ is density function of random variable Z and $F_3(z)$ is its distribution function; α_1, α_2, α_3 and α_4 are coefficients determined on the basis of experimental data in the way assuring following assumptions to be fulfilled [4] for each x_1, x_2, x_3 being from the range $|x_i|\le 1$:

$$1+\alpha_1 x_1 x_2+\alpha_2 x_1 x_3+\alpha_3 x_2 x_3\ge 0 \quad (4)$$

$$1+\alpha_1 x_1 x_2+\alpha_2 x_1 x_3+\alpha_3 x_2 x_3+\alpha_4 x_2 x_2 x_3\ge 0 \quad (5)$$

(c) By using the method of ordinary kriging.

In previous works, the method of ordinary kriging was applied for the one-dimensional case [5] and two-dimensional case, but by determined value of one variable [6] (the direction of influence was parallel to one of the axes). In this paper, the two-dimensional case will be the subject of application, assuming any direction of influence.

2. Theoretical Basis

Let P_1, …, P_n be the measuring points, where $P_i = (\rho_i, d_i)$ and $Z(P_i)$ is the ash contents in particle from point P_i. The goal is to evaluate the ash contents in the point $P_0 = (d_0, \rho_0)$, so to determine the value $Z(P_0)$.

As the estimator of $Z(P_0)$, Eq. (6) will be accepted [7]:

$$Z^*(P_0)=\sum_{k=1}^{n}\lambda_i Z(P_i) \quad (6)$$

where $\sum_{k=1}^{n}\lambda_k=1$.

The values λ_k are selected in the way assuring the function

$$H(\lambda_1,\lambda_2,\ldots,\lambda_n)=E\left[\left(Z(P_0)-\sum_{k=1}^{n}\lambda_i Z(P_i)\right)^2\right] \quad (7)$$

obtaining minimum by the condition $\sum_{i=1}^{n}\lambda_i=1$, where $E(u)$ is the mean value of random variable U.

Applying the Lagrange function

$$L(\lambda_1,\lambda_2,\ldots,\lambda_n,\mu)=H(\lambda_1,\lambda_2,\ldots,\lambda_n)+\mu\left(\sum_{k=1}^{n}\lambda_i-1\right) \quad (8)$$

as well the condition that the conditional extreme of this function exist it can be obtained that the multipliers $\lambda_i(I = 1, \ldots, n)$ and the coefficient μ fulfill the Eq. (9).

$$G\cdot U=G_1 \quad (9)$$

where

$$G=\begin{bmatrix}\gamma(h_{11}) & \gamma(h_{12}) & \ldots & \gamma(h_{1n}) & 1\\ \gamma(h_{21}) & \gamma(h_{22}) & \ldots & \gamma(h_{2n}) & 1\\ \vdots & & & & \vdots\\ \gamma(h_{n-1,1}) & \gamma(h_{n-1,2}) & \ldots & \gamma(h_{n-1,n}) & 1\\ \gamma(h_{n1}) & \gamma(h_{n2}) & & \gamma(h_{nn}) & 0\end{bmatrix} \quad (10)$$

$$X=\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n\\ \mu\end{bmatrix},\ G_1=\begin{bmatrix}\lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n\\ \mu\end{bmatrix} \quad (11)$$

H_{ij} is the distances between measuring points P_i and P_j and h_{i0} is the distance between points P_0 and P_i; i, j = 1, 2, …, n.

Function $\gamma(h)$ is the variogram function. The following variogram models will be taken into consideration in this paper:

Spherical model

$$\gamma(h)=\begin{cases}0 & for\quad h=0\\ c\left(\frac{1.5}{a}h-0.5\frac{h^3}{a^3}\right)+c_0 & for\ h\in(0,a]\\ c+c_0 & for\quad h>a\end{cases} \quad (12)$$

Exponential model

$$\gamma(h)=ce^{ah} \quad (13)$$

Exponential model

$$\gamma(h)=cx^a \quad (14)$$

Linear model

$$\gamma(h)=ax+b \quad (15)$$

As the estimator of variogram function, the Matheron estimator can be applied [7-9], which formulae is:

$$\tilde{\gamma}(h)=\frac{1}{2N(h)}\sum_{i=1}^{N(h)}\left(Z(P_i+h)-Z(P_i)\right)^2 \quad (16)$$

where, $P_i + h$ is the point distanced from P_i by value h, $N(h)$ is the number of combinations of pairs of points (P_i, P_j), which are distanced from themselves by value h.

3. Application

For practical illustration of ordinary kriging method, the grained material of energetic hard coal type 31 was classified and for 72 measuring points $P_i = (d_i, \rho_i)$ the diameter and density were measured, where d_i is the mean particle size given in mm, ρ_i is particle density given in g/cm^3 and $Z(P_i)$ is ash contents given in %.

For assuring equal influence of both characteristics on distance between points, the transformation of random variables was done, $u = \alpha d$, $v = \beta\rho$, where α is numerical coefficient and β is given in $\left(\frac{cm^3 mm}{g}\right)$. In this case, it was accepted that $\alpha = 1$, $\beta = 10$.

The measuring results are given in Fig. 1.

As the distance between points P_i and P_j the following formulae was accepted

$$d(P_i, P_j) = \sqrt{u_i^2 + v_i^2} = \sqrt{\alpha^2 d_i^2 + \beta^2 \rho_i^2} \tag{17}$$

The point, for which the ash contents will be calculated, will be P_0 (9, 15). As the range of influence, the circle $K(P_0, r)$ of the middle P and radius $r = 3$ was accepted.

The points P_i, $i = 1, 2, \ldots, 12$, influence on the value $Z(P_0)$.

In purpose of fitting variogram function by least squared method, the values of coefficients occurring in individual models were selected. The pairs of points for which the distances were similar were classified into the same group $N(h)$.

The following forms of variograms were given:

Linear model:

$$\gamma_1(h) = 50.26h - 33.06 \tag{18}$$

Exponential model (1)

$$\gamma_2(h) = 11.12e^{0.665h} \tag{19}$$

Spherical model

$$\gamma_3(h) = \begin{cases} 173\left(0.33h - 0.006h^3\right) & for\ h \le 2\sqrt{5} \\ 166.34 & for\ h > 2\sqrt{5} \end{cases} \tag{20}$$

Exponential model (2)

$$\gamma_4(h) = 18.73h^{1.6} \tag{21}$$

The values of variograms for individual models and empirical variogram are given in Table 1.

Because none of the models gave the satisfying approximation of empirical variogram $\tilde{\gamma}(h)$, the combined model was applied in form

$$\gamma(h) = \begin{cases} 0 & for \quad h = 0 \\ 8.15e^{0.9056h} & for \quad h \in (0,3] \\ 64.79h - 70.52 & for\ h \in \left(3, \sqrt{13}\right) \\ 179.48\left(0.33 - 0.006h^3\right) & for \quad \left[\sqrt{13}, 4.3\right] \\ 169.07 & for \quad h > 4.3 \end{cases} \tag{22}$$

The values of variogram $\gamma(h)$ are given in Table 2 and Fig. 2.

To evaluate the obtained results, the mean standard error was used which is given by the equation

$$s_r = \sqrt{\frac{\sum_i \left(\gamma_i(h) - \tilde{\gamma}_i(h)\right)^2}{n-2}} \tag{23}$$

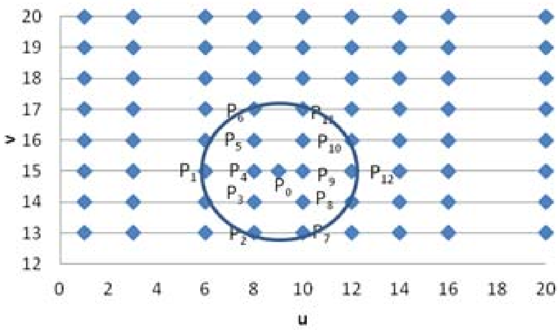

Fig. 1 Locations of empirical points.

Table 1 Values of empirical and theoretical variograms.

h	$\tilde{\gamma}(h)$	$\gamma_1(h)$	$\gamma_2(h)$	$\gamma_3(h)$	$\gamma_4(h)$
1	20.65	17.2	21.43	56.66	18.73
2	49.11	64.46	41.67	107.61	56.98
3	122.77	117.12	81.02	145.84	108.03
$\sqrt{13}$	163.62	148.16	121.20	160.31	145.78
4	170.89	167.38	157.54	165.39	172.12
$2\sqrt{5}$	173.25	191.71	215.65	166.34	205.76
6	173.72	268.5	295.68	166.34	300.29

Table 2 Values of empirical and combined variograms.

H (15)	$\tilde{\gamma}(h)$	$\gamma(h)$
1	20.65	20.16
2	49.11	49.85
3	122.77	123.85
$\sqrt{13}$	163.62	163.08
4	170.89	167.99
$2\sqrt{5}$	173.25	168.56
6	173.72	169.07

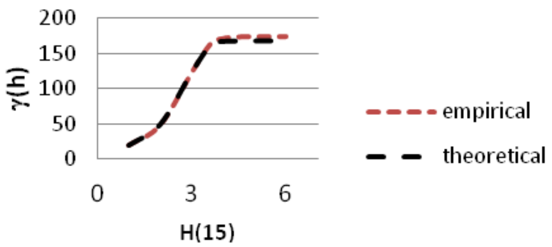

Fig. 2 Graphical interpretation of given variograms.

Table 3 Comparison of values of $Z(P_i)$ given empirically and by ordinary kriging method.

P_i	$\widetilde{Z}(P_i)_{emp}$	$Z(P_i)_{theor}$
1	14.84	15.93
2	10.70	13.47
3	12.09	13.98
4	16.20	19.56
5	25.05	26.69
6	34.50	38.37
7	6.42	6.47
8	16.21	16.71
9	21.58	23.02
10	30.33	29.01
11	32.14	34.33
12	23.18	21.39

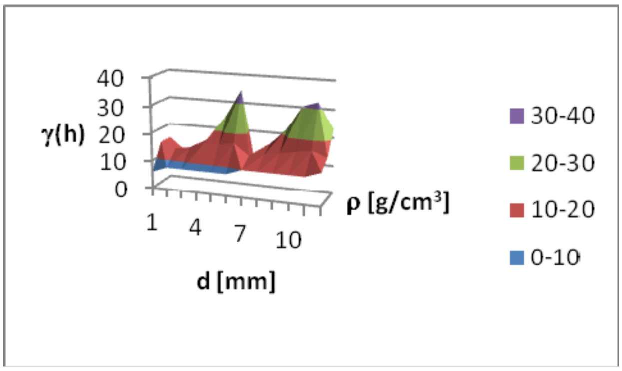

Fig. 3 Empirical three-dimensional plot of $\gamma(h)$ versus d and ρ.

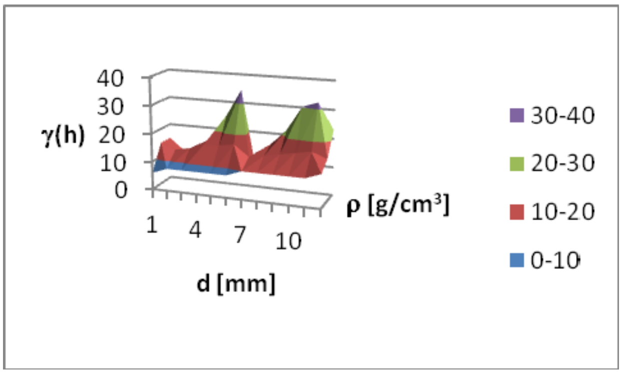

Fig. 4 Theoretical three-dimensional plot of $\gamma(h)$ versus d and ρ.

In this case $s_r = 3.02$.

To determine the values of ash contents in P_0, the surround $K(P_0, r)$ was selected, where $r = \sqrt{5}$.

By solving the set of Eq. (9) for points P_3, P_4, P_5, P_8, P_9, P_{10} and using the calculated values of λ_i in Eq. (6), it occurs that

$$Z(P_0) = -0.01156Z(P_3) + 0.52312Z(P_4) - 0.01156Z(P_5)$$
$$- 0.0115619 \ Z(P_6) = 18.80$$

To verify the accepted model, the values of $Z(P_i)$ were calculated in points which were the basis for calculating variogram, taking points P_j to estimation which are located within $K(P_i, \sqrt{5})$ without point P_i.

The approximation results are given in Table 3 and their interpretation is presented in Figs. 3 and 4.

4. Conclusions

The application of ordinary kriging method to interpolate the values of ash contents in coal dependably on particle size and particle density proved that this method can be used not only in traditional geostatistics but also for other purposes. The obtained results confirm the proper selection of the variogram model for this area as well good selection of range of activity of chosen model.

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