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Estimating Smoothed GDP Components*

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Eurocoin indicator is published monthly by Bank of Italy and CEPR: it provides a summary index of the medium to long-run component of the growth (MLRG), that is the smoothed component of the GDP growth rate. The innovation of this research is some procedures, based on Eurocoin approach and generalized dynamic factor model, that are tested to estimate MLRG for GDP components (consumption, investment, exports, imports) concerning Euro Area aggregate and some specific national case. Results concerning European aggregate show that it is not always clear if it is more useful to use national or European variables to project Expenditure Components by dynamic factor model: It is proposed to test different strategies.

Keywords: band-pass filters, smoothed growth for GDP components, real time performance

Introduction

In this paper medium to long run growth rate (MLRG) for GDP components in Euro Area is outlined. It can be very useful to estimate GDP components (consumption, investment, exports, imports), because their national flash estimate is released by Eurostat about ten weeks after the end of the reference quarter.

New Eurocoin is a synthetic and up-to-date statistics measure of the Euro-Area conjuncture, with releasing several months ahead of the official Euro-area GDP estimate. The target of NE is c_t, the medium to long-run component of the Euro-area GDP growth. It is a performance measure published monthly by the Bank of Italy and CEPR. Estimations of NE are obtained through the generalized dynamic factor model. The specific references are Altissimo et al. (2001), Altissimo, Cristadoro, Forni, Lippi, and Veronese (2009), Forni, Hallin, Lippi, and Reichlin (2001, 2004, 2005).

In the dynamic factor model approach, a vector of *n* time series is decomposed into two mutually orthogonal components: a common component characterized by few common factors or latent shocks, and a component "idiosyncratic", led by *n* specific shocks (one for each variable in the panel). These models allow a net reduction of cross-sectional size of the dataset. It is a well-known result in the literature isolating the business cycle in integrated series that band-pass filter could deteriorate at the end of the sample. Altissimo, Cristadoro, Forni,

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Lippi, and Veronese (2008) show that the same problem arises with application to stationary time series. And, through New Eurocoin, they develop a method to obtain smoothing of a stationary time series so as to avoid the occurrence of end-of-sample deterioration.

New Eurocoin (NE) is the projection of the bandpassed GDP on a set of regressors, which are linear combinations of the variables contained in the Thomson Financial Datastream used by the Bank of Italy. It produces real-time monthly estimate of GDP growth, purified from erratic components (short-run fluctuations). Band-pass filters can eliminate erratic components as they are infinite moving averages and are based on past and future values of GDP. However, unlike Eurocoin, they are less reliable for the most recent data, very relevant for economic policy.

In this paper, by using Eurocoin methodology, some approaches and results are shown to obtain smoothing of a stationary time series so as to avoid the occurrence of end-of-sample deterioration, with the objective to build real-time monthly estimates of GDP components growth, purified from erratic components (short-run volatility, measurement errors, seasonal variations).

The innovation of Eurocoin with respect to the econometric literature is a procedure to remove both the idiosyncratic ad the short-run components, so that the resulting factors are both common and smooth.

Estimates of GDP Components

In this section the paper outlines indicators explaining the medium to long run component (MLRG) of GDP useful to analyze characteristics of GDP components growth rates. This goal is achieved by the generalized dynamic factor model (actually used to calculate Euro MLRG by Eurocoin index). Disaggregated models built in this paragraph are tested also by real time simulations in next section.

Data

Data and variables contained in Thomson Financial Datastream, used in this research, are the following:

This study focus on the medium to long-run component of the growth (MLRG), that is the smoothed component of the GDP growth rate obtained by removing the fluctuations of period shorter than or equal to one year, and it bears no relationship to any definition of trend.

Model and Strategies

In a classic dynamic factor model, considering the scalar time series variable Y_t to forecast and let X_t be the N-dimensional time series of candidate predictors, it is assumed that (X_t, Y_{t+h}) admits a factor model with *r* common latent factors F_t:

$$X_t = \Lambda F_t + \varepsilon_t \tag{1}$$

$$Y_{t+h} = \beta_F' F_t + \beta_\omega' \omega_t + \varepsilon_{t+h} \tag{2}$$

where ε_t is an $N \times 1$ vector of idiosyncratic disturbances, h is the forecast horizon, ω_t is an $m \times 1$ vector of observed variables (e.g., lags of Y_t) useful, with F_t, to forecast Y_{t+h}. The value of c_t, with the coefficients A_i, at the end of the sample is so estimated:

$$\hat{c}_T = A_1 F_{1T} + A_2 F_{2T} + ... A_m F_{mT} \tag{3}$$

In our empirical disaggregation, it is possible to obtain different indicators, by the strategies (methods) below indicated:

(1) Projection of Euro Area GDP components (Consumptions, Investments, Foreign Trade) on European factors, to estimate Euro Area Smoothed Expenditure Components. European factors are combinations of the 157 variables contained in the Thomson Financial Datastream (see Tables 1 and 2) and used by the Bank of Italy to build Eurocoin.

(2) Comparing the Italian case to the one concerning Germany, to estimate National Expenditure Components the following strategies can be used:

- Projection of National GDP components on National Factors (e.g., projection of German Household Consumptions on common factors built by the 39 German variables contained in our dataset, see Table 2);
- Projection of National GDP components on European factors (e.g., projection of Italian Consumptions on the European factors).

Table 1

List of Variables

Data source	Variables
Surveys	31
Leading indicators	6
Demand indicators	12
Industrial production	32
Wages indicators	2
Employment indicators	5
Producer price index	26
Exchange rates	3
Imports-exports	8
Money supply	8
Standard & poor's index	7
(Italy, Germany, USA, UK) SPREAD	10
Benchmark bond	7
Total	157

Table 2

National Variables Contained in Thomson Datastream

Geographic area	Variables
Belgium	14
Finland	2
France	23
Germany	39
Greece	1
Italy	22
Netherlands	5
Spain	25
UK	4
USA	7
Euro area	15
Total	157

In Tables 3 and 4 that follow, based on in sample data and real time performance, our results are shown for macroeconomic variables that influence the classical equation of gross domestic product in National Accounting:

$$GDP = Cons + Invest + Var.\ Stocks + Export - Import$$

Eurocoin is obtained as a projection of the bandpassed GDP on factors by a generalization of the principal components concept. Each series contained in the dataset for the calculation of Eurocoin is obtained as follows:

$$x_{it} = \chi_{it} + \xi_{it} = b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + ... + b_{iq}(L)u_{qt} + \xi_{it}$$

with a common component χ and an idiosyncratic ξ, that are orthogonal at all lead and lags. The common components χ in the classical model of dynamic factors have the following form:

$$\chi_{it} = c_{i1}F_{1t} + c_{i2}F_{2t} + ... + c_{ir}F_{rt}$$

The value of c_t, with the coefficients A_i, at the end of the sample is so estimated:

$$\hat{c}_T = A_1F_{1T} + A_2F_{2T} + ... A_mF_{mT}$$

The main innovation of our procedure, differently from Eurocoin, is the following: our monthly estimation concerning medium to long-run component of GDP growth rate $\hat{c}^s_T$, will be obtained projecting bandpassed components of GDP (and not the whole GDP as for Eurocoin Indicator), on common smoothed factors :

$$\hat{c}^s_T = A^s_{1F}F_{1t} + A^s_{2F}F_{2t} + ... + A^s_{mF}F_{mT}$$

Estimating Euro Area Expenditure Components of GDP

This section is developed in sample (ex post) estimates, being generated for the same dataset that was used to estimate the model's parameters: This is the reason for which one would expect that these estimates are relatively good (particularly in terms of estimate error), depending also on the volatility concerning the analyzed period; after our in sample estimates are tested by real time performance that is a sensible approach to examine estimate accuracy: not all the observations are used in estimating the model parameters, it will be held some observation back, and the end of the sample (the latter sample) will be used to build pseudo estimates. Comparison among in sample and real rime performance will be useful to value the goodness of our estimates.

In Tables 3-4 and Figures 1-4, they are analyzed both correlation and RMSFE among national bandpassed growth (built by Baxter-King filter) and National Indicators (built by dynamic factor model). The goodness of our estimates is assessed both by in sample (ex post) estimates and by pseudo simulations.

Table 3

Correlation Among Indicators and Bandpassed GDP Components

Variables	June, 1995; December, 2002 (In sample estimates)	January, 2003; September, 2010 (Real time estimates)
Final Consumptions	0.74	0.80
Household Cons.	0.77	0.85
Government Expend.	0.65	0.31
Investments	0.89	0.55
Exports	0.89	0.84
Imports	0.96	0.91

Table 4

RMSFE Among Indicators and European Bandpassed GDP

Variables	June, 1995; December, 2002 (In sample estimates)	January, 2003; January, 2008 (Real time estimates)	January, 2003; September, 2010 (Real time estimates)
Final Consumptions	0.13	0.13	0.20
Household Cons.	0.16	0.16	0.25
Government Expend.	0.14	0.14	0.18
Investments	0.49	0.49	-
Exports	0.57	0.57	1.49
Imports	0.35	0.49	1.21

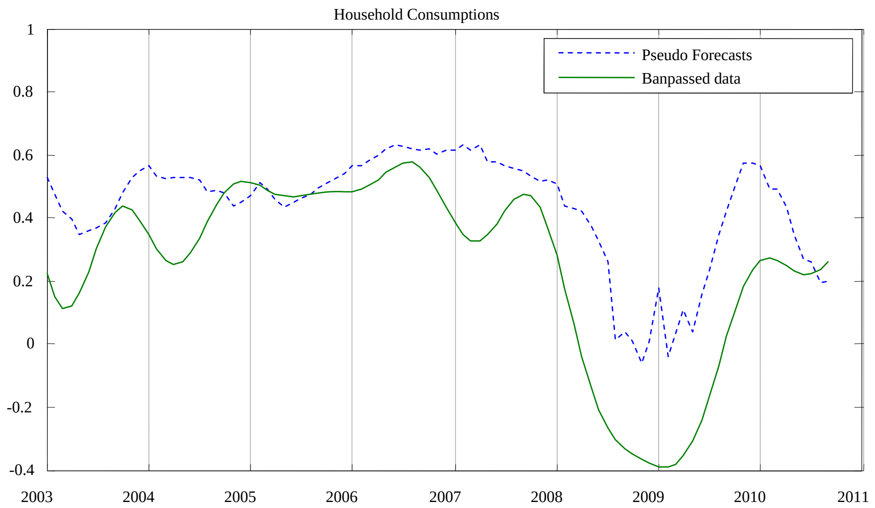

Figure 1. Real time performance versus banpassed data: Consumptions.

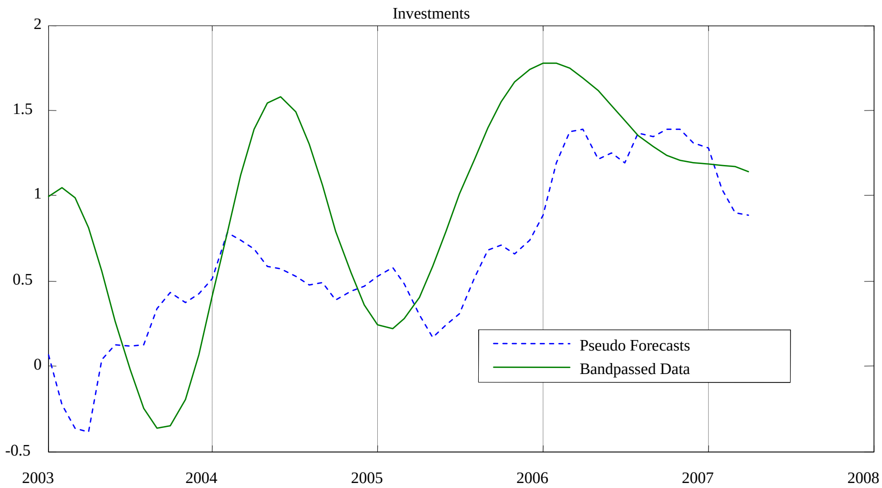

Figure 2. Real time performance versus banpassed data: Gross capital formation.

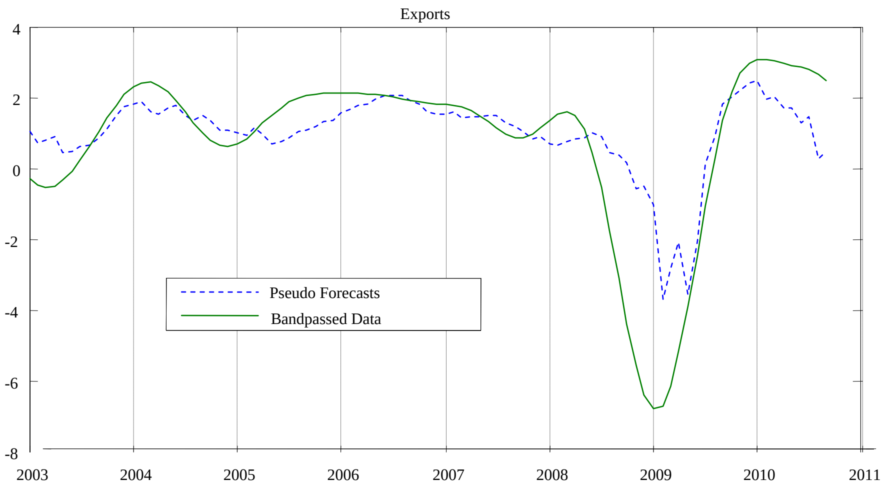

Figure 3. Real time performance versus banpassed data: Export trade.

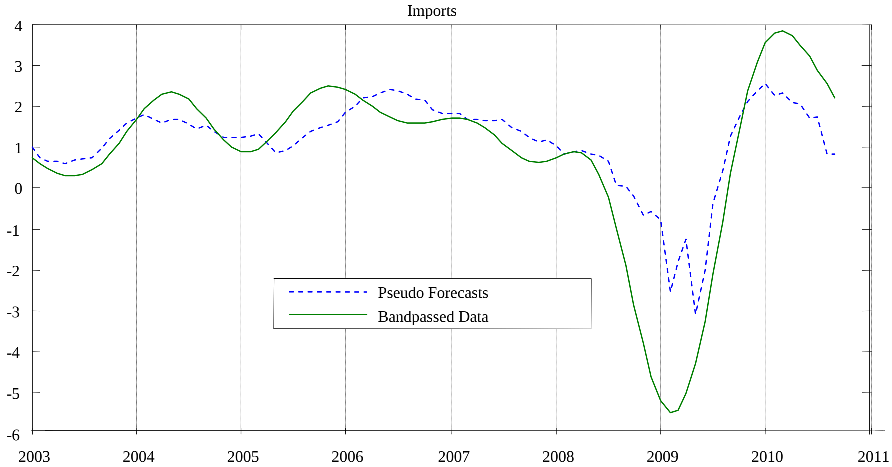

Figure 4. Real time performance versus banpassed data: Import trade.

The period 1995-2010 is investigated in this paper to analyze business cycles. Real time estimates are built from 2003 to 2008 and from 2003 to 2010, separately, because in 2008-2010 it is observed a strong recession and an high variation in volatility concerning GDP.

Among the Euro Area GDP components estimated in real time in this section this paper observes that:

- In terms of correlation and RMSFE both Final Consumptions and Household Consumptions have been particularly well estimating smoothed expenditure components in the recent past;
- Estimates for government expenditure are relatively bad in terms of correlation;
- Concerning foreign trade (Exports, Imports) is possible to observe the goodness of our pseudo estimates in terms of correlation, but RMSFE appears particularly high;
- Also for Investment RMSFE appears quite high;
- Different performances from in sample estimates and out of sample period are also influenced on the volatility in the relative periods.

Then, the 157 European variables from Thomson Financial Datastream, used to build our common factors, are particularly useful to build consumption indicators, but their whole contribution is less clear for foreign trade and investments. The figures below (concerning data outlined in Tables 3 and 4) show, for each GDP component, pseudo real time estimates and their comparisons with the relative banpassed components.

It is also tested to project Export bandpassed data only on the 11 variables concerning Foreign Trade contained in Thomson Financial Datastream; in this case no improvement has been obtained in terms of RMSFE, and correlation decreases at 0.46 (during 2003-2008) .

Estimating National Household Consumptions

In this section they are outlined some national indicators explaining the medium to long run component (MLRG) of GDP, useful to analyze characteristics of GDP components growth rates. Household Consumptions are projected both on European factors and on national factors, to estimate monthly National Consumptions: this study compares the Italian case to the one concerning Germany. In Tables 5 and 6 and Figure 5, correlation and RMSFE are analyzed, for every country, between national bandpassed growth (built by Baxter-King filter) and National Indicators built by dynamic factor model.

Table 5

The Italian Case: Correlation and RMSFE Among Banpassed Components and Expenditure Indicators

Strategies	June, 1995; December, 2002 (In sample estimates)	January, 2003; September, 2010 (Real time estimates)
Projection of National Household Consumptions on European Variables	0.76 (0.29)	0.81 (0.23)
Projection of National Household Consumptions on National Variables	0.88 (0.21)	0.83 (0.19)

Note. RMSFE is indicated in parenthesis.

Table 6

The German Case: Correlation and RMSFE Among Banpassed Components and Expenditure Indicators

Strategies	June, 1995; December, 2002 (In sample estimates)	January, 2003; September, 2010 (Real time estimates)
Projection of National Household Consumptions on European Variables	0.57 (0.34)	Correlation lower than 0.15 (0.38)
Projection of National Household Consumptions on National Variables	0.69 (0.30)	Correlation lower than 0.15 (0.39)

Note. RMSFE is indicated in parenthesis.

If we analyze the ability of the real time indicator $\hat{c}_t^{\,z}$ both for Italian and German Consumptions, with $z = 1,2,$ in estimating the approximate truncated band-pass filter $c_t^{*z}(T)$, we can calculate the ability of $\hat{c}_t^{\,z}(t) - \hat{c}_{t-1}^{\,z}(t) = \Delta\hat{c}_t^{\,z}(t)$ to signal the correct change of the bandpassed variation $\Delta c_t^{*z}(T)$.

Therefore, we analyze the performance at time t, with $t \le T - 12$, by the difference between our indicator at time t and the approximate target at t that is obtained using data up to T. Following Pesaran and Timmermann (1992), we observe in terms of sign prediction of the bandpassed target that a national indicator in real time concerning Italy provides a satisfactory approximation of band-pass Consumptions, in particular by the projection of National Consumptions on the 22 Italian variables included in the dataset (0.59% of correct prediction in the period 2003-2009 against a 0.52 obtained by projecting National Consumptions on the 157 European variables).

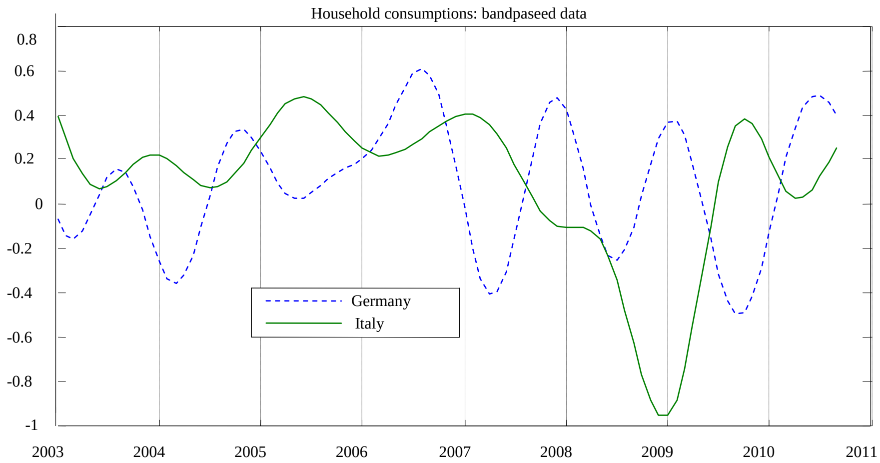

Figure 5. Bandpassed consumptions in Italy and Germany.

Main Conclusions

In this paper are performed and tested some methods to estimate medium to long run component of the growth for expenditure components concerning some countries (Germany and Italy) belonging to Euro Area and the European aggregate. Firstly, following the literature concerning bandpass filters (see e.g. Baxter and King, 1999, and Christiano and Fitzgerald, 2003) to obtain estimates of smoothed growth rates. Thomson Financial Datastream has been used to strengthen the validity of the exercise and to build our estimates.

Secondly, two different methods are applied, to project bandpassed component of the growth by generalized dynamic factor model. From the results achieved, our main findings can be so summarized:

- To estimate Euro Area Smoothed Household Consumptions, European common factors that are combinations of the 157 variables contained in the Thomson Financial Datastream are very useful to obtain a real-time, monthly estimate of area-wide Consumption growth rate ;
- In terms of RMSE and prediction signs the national indicator concerning Italy provide a good approximation of band-pass Consumptions in real time, in particular by the projection of National Consumptions on the 22 Italian variables included in the dataset: this strategy seems to maximize the capacity of our indicator to estimate MLRG at the end of the sample;
- Concerning foreign trade this study shows the goodness of the pseudo real time estimates in terms of correct sign prediction and turning points for the Import variable, but RMSE appears particularly high.

Results concerning Italy and European aggregate show that it is not always clear if it is more useful to use national or European variables to project Expenditure Components by a dynamic factor model. Clearly, such conclusions are not general but restricted to the data and the models used in this exercise.

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