

Labour Productivity and the Chaotic Economic Growth Model: G7

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Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Almost three years after the crisis, the G7 countries continue to be challenged with economic volatility. The global economy has slowed. Growth in the United States has weakened. In Europe, economic instability is generated by the financial and economic imbalances. Europe is gripped with financial strains from the sovereign debt crisis in the euro area periphery. How these G7 economies confront their fiscal challenges will profoundly affect their economic stability. The basic aim of this paper is to provide a relatively simple chaotic economic growth model that is capable of generating stable equilibria, cycles, or chaos. This paper looks in more detail at the GDP growth stability issues in each of the G7 countries in the period 1990-2012 (Retrieved from <http://www.imf.org>). A key hypothesis of this work is based on the idea that the coefficient $\pi = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right]$ plays a crucial role in explaining local stability of the gross domestic product growth, where, p —the coefficient of labour productivity; p_m —the coefficient of the marginal labour productivity, s_p —private saving rate; i —investment rate; b —percent of the gross domestic product which belongs to budget deficit; b_m —marginal budget deficit coefficient; n —net capital outflow rate.

Keywords: stability, budget deficit, labour productivity, the gross domestic product, chaos

Introduction

Chaos theory attempts to reveal structure in aperiodic, unpredictable dynamic systems. Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (1) extreme sensitivity to initial conditions; (2) cause and effect are not proportional; and (3) nonlinearity.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1997), Gandolfo (2009), Grandmont (1985), Goodwin (1990), Medio (1993, 1996), Lorenz (1993), Jablanovic

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(2010, 2011, 2012), among many others.

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National saving is the source of the supply of loanable funds. Domestic investment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium interest rate, the amount that people want to save exactly balances the amount that people want to borrow for the purpose of buying domestic capital and foreign capital.

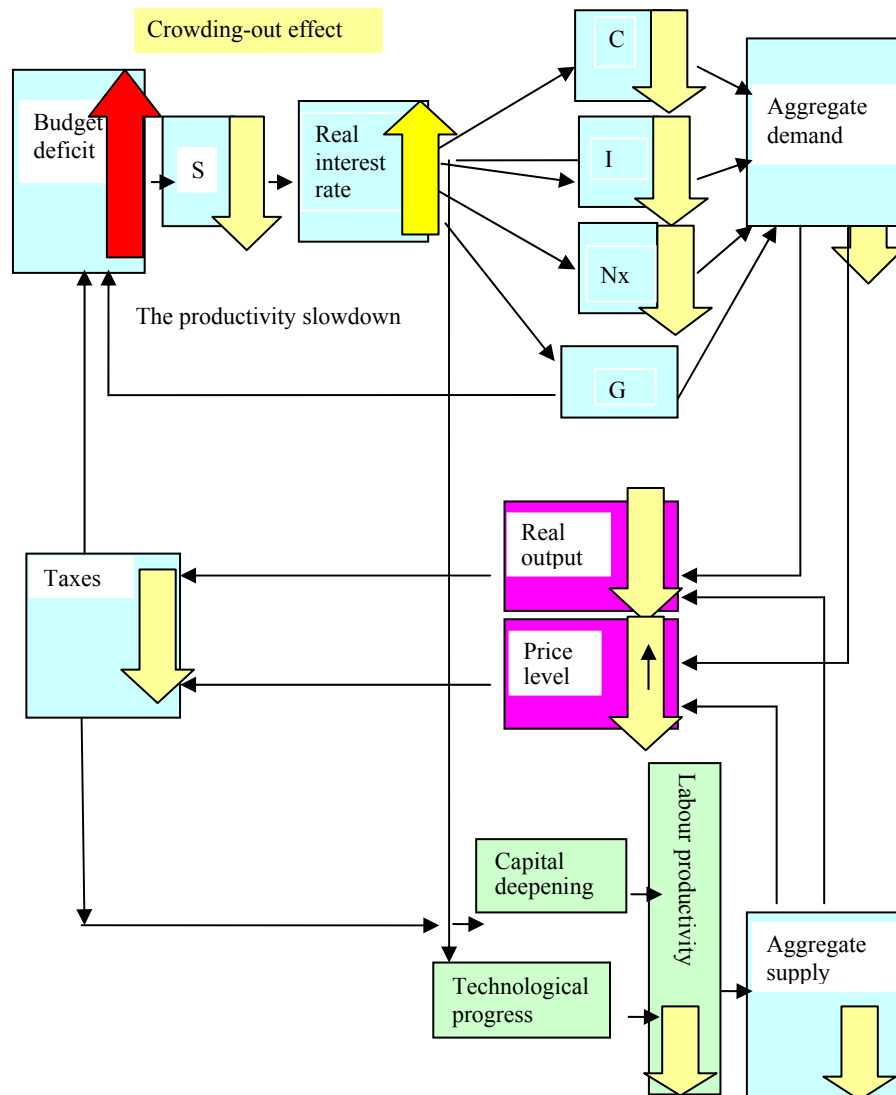


Figure 1. The effects of a government budget deficit. When the government runs a budget deficit, it reduces the national saving. The interest rate rises. The higher interest rate reduces investment and labour productivity growth. Further, the higher interest rate reduces net foreign investment (net capital outflow). The real gross domestic product falls as a consequence of the negative open economy multiplier effects and the negative investment multiplier effects.

In an open economy, government budget deficit, as a negative national saving, raises real interest rates, crowds out domestic investment, decreases net capital outflow, causes the domestic currency to appreciate.

However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, net exports fall. Further, the real gross domestic product falls as a consequence of the negative open economy multiplier effects.

Almost three years after the crisis, the G7 countries continue to be challenged with economic volatility. The global economy has slowed. Growth in the United States has weakened. In Europe, economic instability is generated by the financial and economic imbalances. Europe is gripped with financial strains from the sovereign debt crisis in the euro area periphery. How these G7 economies confront their fiscal challenges will profoundly affect their economic movements.

The G7 countries have experienced economic and financial turbulence. High budget deficits and debt, lower national saving, an decrease in labour productivity growth rate between 2000 and 2010, and tensions between saving and investment are weighing on growth in much of the G7 economies (see Figure 2).

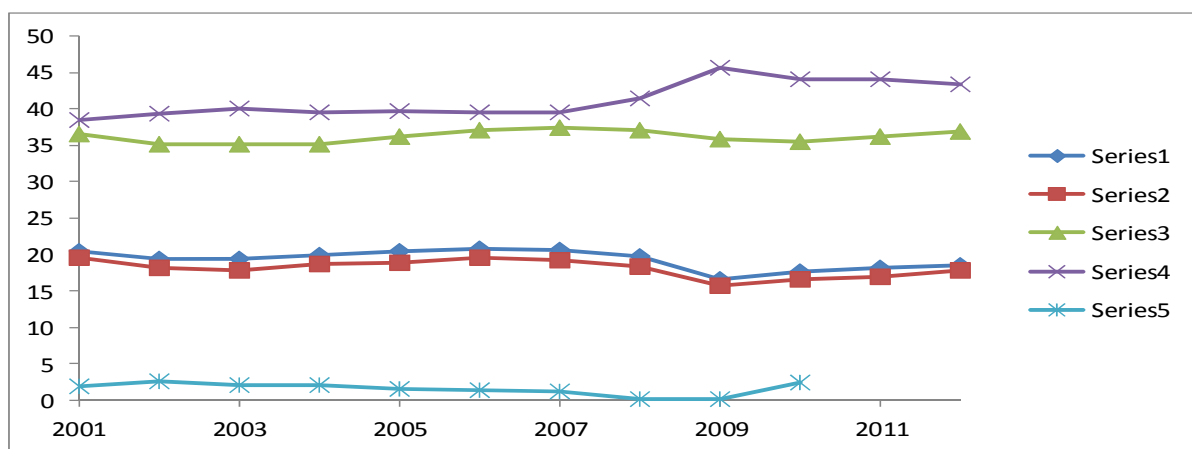


Figure 2. The G7 countries (2001-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (2001-2012) Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (2001-2010).

Also, the G7 countries experienced an decrease in labour productivity growth rate between 2000 and 2010 (see Figure 3 and Figure 4).

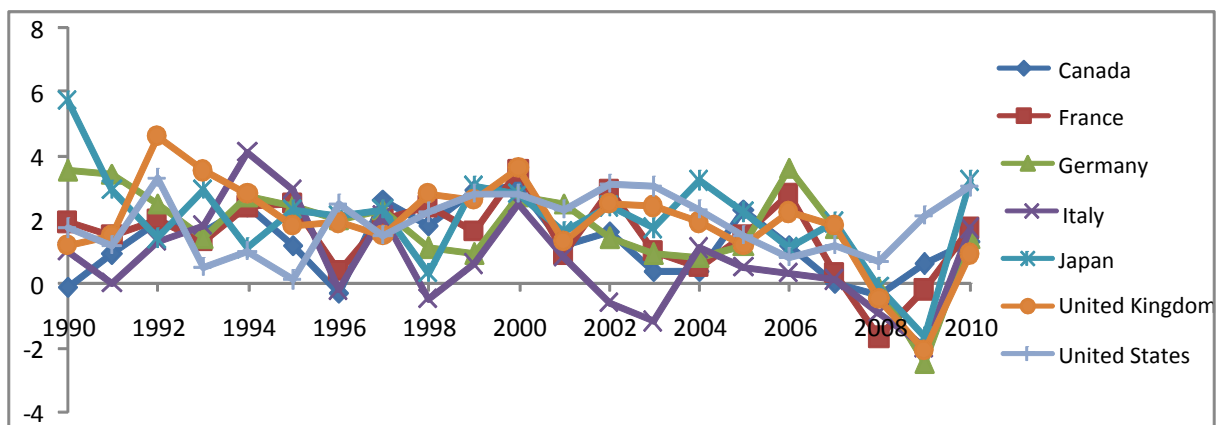


Figure 3. G7 (1990-2010): Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010). Source: Organization for Economic Cooperation and Development (2011).

Labour productivity growth can be attributed to two causes: (1) technological progress; and (2) capital deepening. Capital deepening is the rate of growth of capital per unit of labor input. When the capital-labor ratio is increasing, each worker can increase production because the amount of capital available is growing.

The tendency in labor productivity growth in the G7 countries since the mid-1990s has attracted significant attentions because the labor productivity growth represents an important factor of economic growth stability.

Productivity growth in Europe exceeded that in the United States up to the mid-1990s, but since then European performance has slackened while the United States has picked up and taken a lead. High-tech sectors have played an important role in the acceleration of productivity growth in the United States. Europe did not benefit to the same degree from the productivity boost that came from the information technology revolution. The downturn in labor productivity growth in Europe may also reflect slower growth in capital-labor ratios. Total factor productivity growth in Europe may also have declined, while in the United States it may have increased.

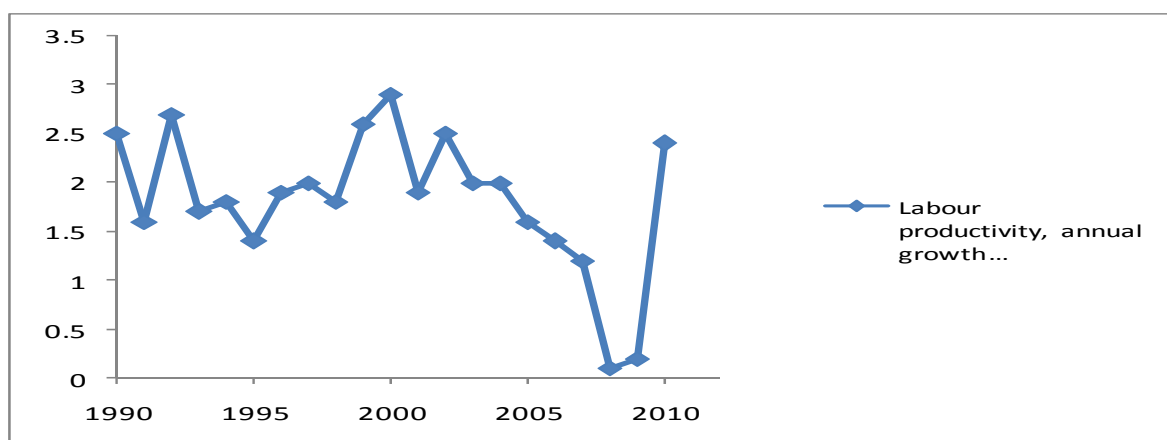


Figure 4. G7 (1990-2010): Labour productivity, annual growth rate. Retrieved from <http://www.oecd.org> (1990-2010). Source: Organization for Economic Cooperation and Development (2011).

It is important to compare (see Figures 5-11) total investment (percent of GDP), gross national saving (percent of GDP), general government revenue (percent of GDP), general government total expenditure (percent of GDP), and labour productivity growth rate in the G7 countries in the period (1990-2012).

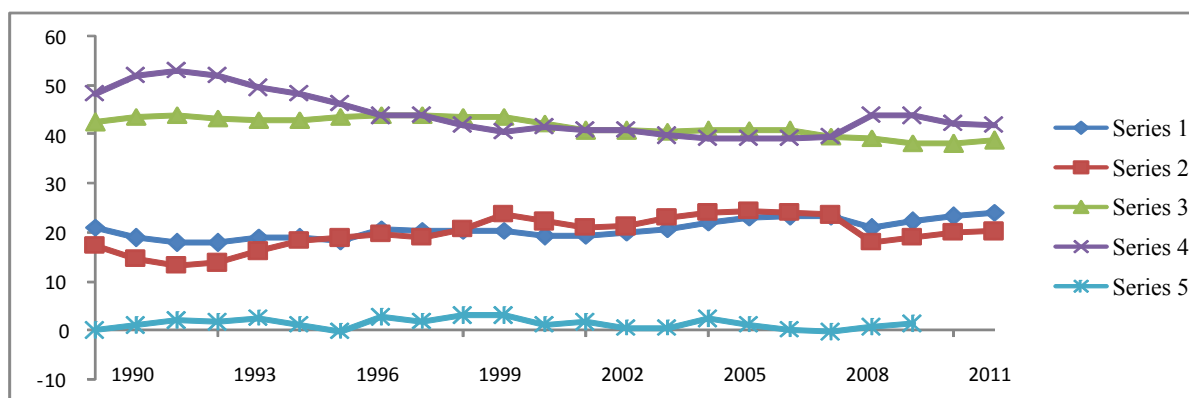


Figure 5. Canada (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP) (1990-2012) Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010).

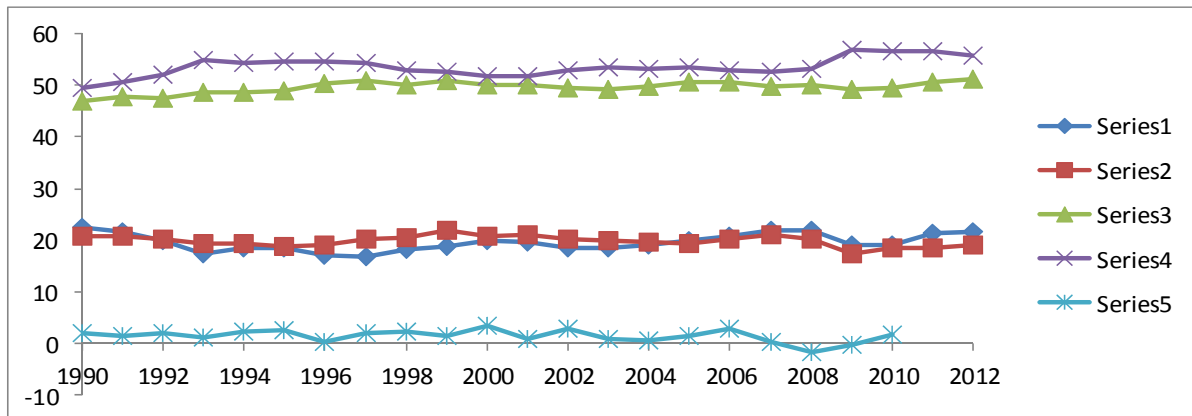


Figure 6. France (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP) (1990-2012), Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010).

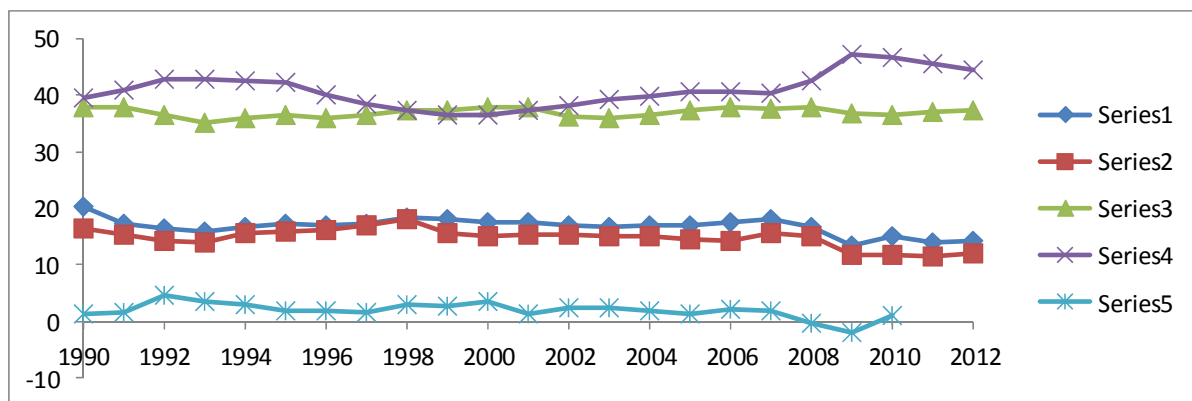


Figure 7. UK (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (1990-2012), Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, www.oecd.org (1990-2010).

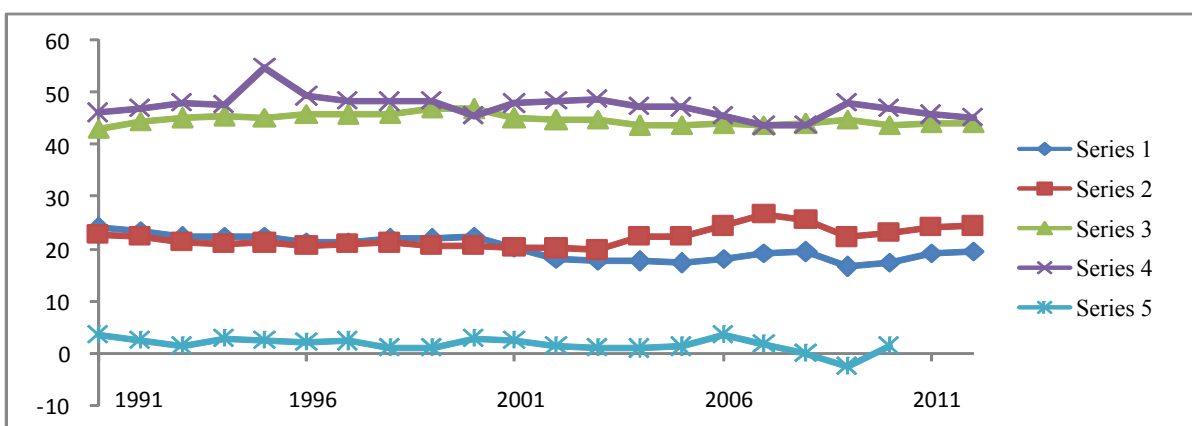


Figure 8. Germany (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (1990-2012) www.imf.org, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010).

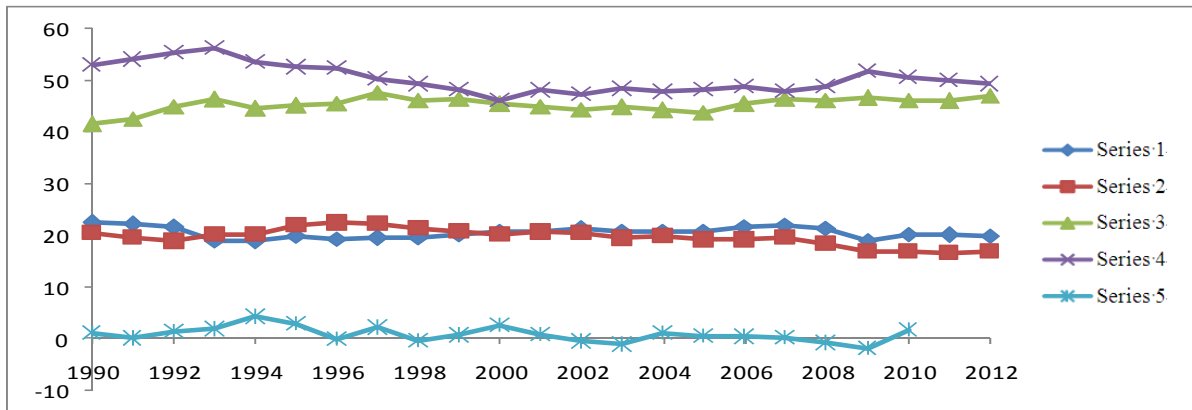


Figure 9. Italy (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (1990-2012), Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, www.oecd.org (1990-2010).

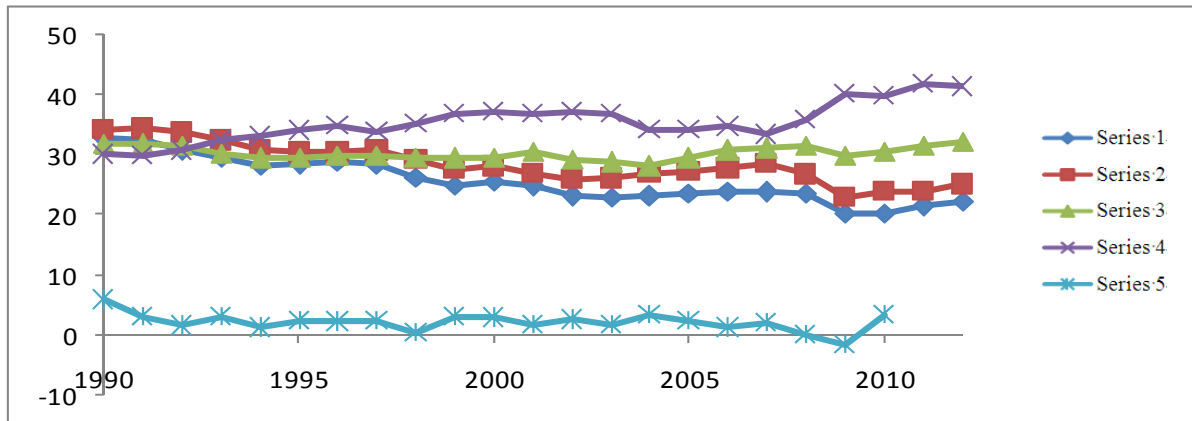


Figure 10. Japan (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (1990-2012), Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010).

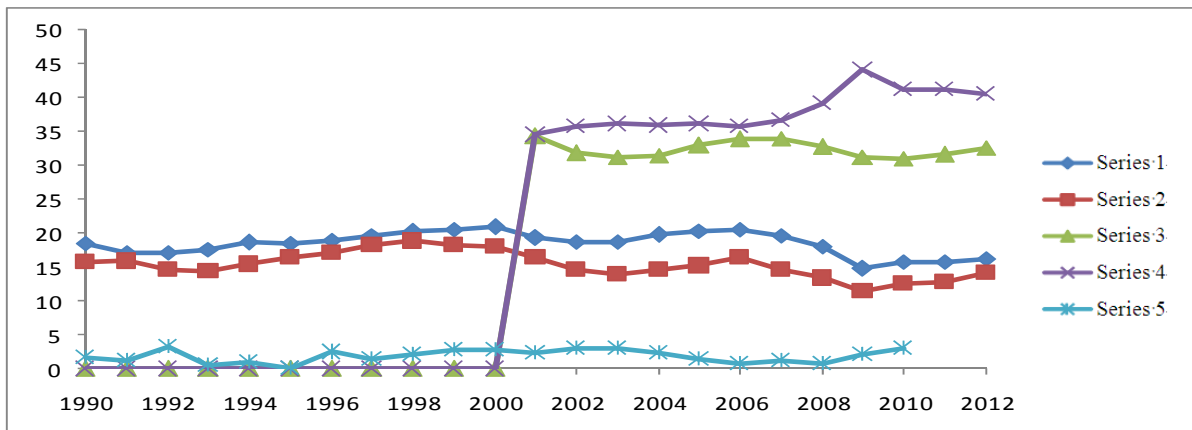


Figure 11. USA (1990-2012): Series 1—Total investment (percent of GDP), series 2—Gross national savings (percent of GDP), series 3—General government revenue (percent of GDP), series 4—General government total expenditure (percent of GDP), (1990-2012), Retrieved from <http://www.imf.org>, series 5—Labour productivity, annual growth rate, Retrieved from <http://www.oecd.org> (1990-2010).

The Model

Irregular movement of the gross domestic product can be analyzed in the formal framework of the chaotic economic growth model:

$$S_t = I_t + NCO_t \quad (1)$$

$$Bd_t = bY_t \quad (2)$$

$$\Delta Bd_t = b_m \Delta Y_t \quad (3)$$

$$S_t = S_{pt} - Bd_t \quad (4)$$

$$S_{pt} = s_p Y_t \quad (5)$$

$$p_m = \frac{\Delta Y}{\Delta L} \quad (6)$$

$$p = \frac{Y_t}{L_t} \quad (7)$$

$$Y_t = L_t^{1/2} \quad (8)$$

$$I_t = i Y_t \quad (9)$$

$$NCO_t = n Y_t \quad (10)$$

where, S : national saving; Y : the real gross domestic product; NCO : net capital outflow (net foreign investment); Bd : budget deficit; S_p : private saving; L : labour; p : the coefficient of labour productivity; p_m : the coefficient of the marginal labour productivity; s_p : private saving rate; i : investment rate; b : percent of the gross domestic product which belongs to budget deficit; b_m : marginal budget deficit coefficient; n : net capital outflow rate.

By substitution one derives:

$$Y_{t+1} = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right] Y_t - \left(\frac{p p_m b_m}{p b - p_m b_m} \right) Y_t^2 \quad (11)$$

Further, it is assumed that the current value of the gross domestic product is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the gross domestic product growth rate depends on the current size of the gross domestic product, Y , relative to its maximal size in its time series Y^m . We introduce y as $y = Y/Y^m$. Thus y ranges between 0 and 1. Again we index y by t , i.e., write y_t to refer to the size at time steps $t = 0, 1, 2, 3, \dots$. Now growth rate of the gross domestic product is measured as:

$$y_{t+1} = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right] y_t - \left(\frac{p p_m b_m}{p b - p_m b_m} \right) y_t^2 \quad (12)$$

This model given by equation (12) is called the logistic model. For most choices of p , b , n , i , s_p , b_m , and p_m there is no explicit solution for equation (12). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect—the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0, 4], \quad z_t \in [0, 1] \quad (13)$$

is equivalent to the iteration of growth model (12) when we use the identification:

$$z_t = \left(\frac{p_m b_m}{s_p - i - n} \right) y_t \quad (14)$$

$$\pi = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right]$$

Using equation (14) and equation (12) we obtain:

$$\begin{aligned} z_{t+1} &= \frac{p_m b_m}{(s_p - i - n)} y_{t+1} \\ &= \frac{p_m b_m}{(s_p - i - n)} \left\{ \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right] y_t - \left(\frac{p p_m b_m}{p b - p_m b_m} \right) y_t^2 \right\} \\ &= \left(\frac{p p_m b_m}{p b - p_m b_m} \right) y_t - \left[\frac{p p_m^2 b_m^2}{(s_p - i - n)(p b - p_m b_m)} \right] y_t^2 \end{aligned}$$

On the other hand, using equation (13) and equation (14) we obtain:

$$\begin{aligned} z_{t+1} &= \pi z_t (1 - z_t) \\ &= \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right] \left[\frac{p_m b_m}{(s_p - i - n)} \right] y_t \left\{ 1 - \left[\frac{p_m b_m}{(s_p - i - n)} \right] y_t \right\} \\ &= \left(\frac{p p_m b_m}{p b - p_m b_m} \right) y_t - \left[\frac{p p_m^2 b_m^2}{(s_p - i - n)(p b - p_m b_m)} \right] y_t^2 \end{aligned}$$

Thus, we have that iterating:

$$y_{t+1} = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right] y_t - \left(\frac{p p_m b_m}{p b - p_m b_m} \right) y_t^2$$

is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using $z_t = \frac{p_m b_m}{(s_p - i - n)} y_t$ and $\pi = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right]$.

It is important because the dynamic properties of the logistic equation (13) have been widely analyzed (Li & Yorke, 1975; May, 1976).

It is obtained that for parameter values:

(1) $0 < \pi < 1$ all solutions will converge to $z = 0$;

(2) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on π ,

(3) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1)/\pi$,

(4) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1)/\pi$,

(5) For $3 < \pi < 4$ all solutions will continuously fluctuate;

(6) For $3.57 < \pi < 4$ the solution become “chaotic” which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Empirical Evidence

The main aim of this paper is to analyze the gross domestic product growth stability in the period 1990-2012, in the G7 countries, by using the presented nonlinear, logistic economic growth model (12) or:

$$y_{t+1} = \pi y_t - \nu y_t^2 \quad (15)$$

where y : the gross domestic product ; $\pi = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right]$ and $\nu = \left(\frac{p p_m b_m}{p b - p_m b_m} \right)$.

Firstly, we transform data on the gross domestic product (Retrieved from <http://www.imf.org>) from 0 to 1, according to our supposition that actual value of the gross domestic product, Y , is restricted by its highest value in the time-series, Y^m . Further, we obtain time-series of $y = Y/Y^m$. Now, we estimate the model (15). The results are obtained in the next section.

Table 1

The Estimated Model (15) in the Period 1990-2012

Canada (1990-2012)		$R = 0.99415$ Variance explained: 98.834%	
		π	ν
	Estimate	1.10198	0.91565
	Std.Err.	0.2857	0.33252
	$t(17)$	38.57063	2.75
	p -level	0.00000	0.013563
France (1990-2012)		$R = 0.99102$ Variance explained: 98.212%	
		π	ν
	Estimate	1.09101	0.84130
	Std.Err.	0.3332	0.37089
	$t(17)$	32.074167	2.268304
	p -level	0.00000	0.036628
Germany (1990-2012)		$R = 0.96187$ Variance explained: 92.519%	
		π	ν
	Estimate	1.08704	0.082378
	Std.Err.	0.06696	0.07478
	$t(17)$	16.23511	1.101595
	p -level	0.00000	0.285991
Italy (1990-2012)		$R = 0.96067$ Variance explained: 92.173%	
		π	ν
	Estimate	1.17887	0.184434
	Std.Err.	0.06333	0.68366
	$t(17)$	18.61356	2.69773
	p -level	0.00000	0.15247
Japan (1990-2012)		$R = 0.90207$ Variance explained: 98.007%	
		π	ν
	Estimate	1.15709	0.162077
	Std.Err.	0.10113	0.109869
	$t(17)$	11.44117	1.475188
	p -level	0.00000	0.158442

(Table 1 continued)

UK (1990-2012)		$R = 0.98999$ Variance explained: 76.158%	
		π	ν
	Estimate	1.13081	0.126393
	Std.Err.	0.3559	0.03989
	$t(17)$	31.76887	3.168569
	p -level	0.00000	0.005614
USA (1990-2012)		$R = 0.99242$ Variance explained: 98.49%	
		π	ν
	Estimate	1.1196	0.112431
	Std.Err.	0.03198	0.036418
	$t(17)$	35.01383	3.087237
	p -level	0.00000	0.006685

Note. Source: Retrieved from <http://www.imf.org>.

Conclusion

This paper suggests conclusion for the use of the chaotic economic growth model in predicting the movement of the gross domestic product in the G7 countries. The model (12) has to rely on specified parameters p , b , n , i , s_p , b_m , p_m , and initial value of the gross domestic product, y_0 .

A key hypothesis of this work is based on the idea that the coefficient $\pi = \left[\frac{p(s_p - i - n)}{p b - p_m b_m} \right]$ plays a crucial role in explaining local stability of the gross domestic product growth, where, p —the coefficient of labour productivity, p_m —the coefficient of the marginal labour productivity, s_p —private saving rate, i —investment rate, b —percent of the gross domestic product which belongs to budget deficit, b_m —marginal budget deficit coefficient, n —net capital outflow rate.

An estimated value of the coefficient π was around 1 in the G7 countries in the period 1990-2012. This result confirms stable economic growth in the G7 countries in the observed period. However, if $\pi < 1$, then the gross domestic product will be decreased. However, national saving in the G7 countries has been relatively decreased (the decreasing slope of the saving function). The tendency of decreasing saving rates and relatively decreased savings in the G7 countries represent challenges for economic policies. Also, the G7 countries experienced an decrease in labour productivity growth rate between 2000 and 2010. The budget deficits have risen to high levels. In this sense, it is important to increase labour productivity, national saving, private saving, net outflow capital; and to decrease budget deficit through fiscal and monetary reforms.

References

- Benhabib, J., & Day, R. H. (1981). Rational choice and erratic behaviour. *Review of Economic Studies*, 48, 459-471.
- Benhabib, J., & Day, R. H. (1982). Characterization of erratic dynamics in the overlapping generation model. *Journal of Economic Dynamics and Control*, 4, 37-55.
- Benhabib, J., & Nishimura, K. (1985). Competitive equilibrium cycles. *Journal of Economic Theory*, 35, 284-306.
- Day, R. H. (1982). Irregular growth cycles. *American Economic Review*, 72, 406-414.
- Day, R. H. (1983). The emergence of chaos from classical economic growth. *Quarterly Journal of Economics*, 98, 200-213.
- Day, R. H. (1997). Complex economic dynamics volume I: An introduction to dynamical systems and market mechanism. In *Discrete dynamics in nature and society* (pp. 177-178), 1. MIT Press.
- Gandolfo, G. (2009). *Economic dynamics* (4th ed.). Berlin: Springer-Verlag.
- Goodwin, R. M. (1990). *Chaotic economic dynamics*. Oxford: Clarendon Press.
- Grandmont, J. M. (1985). On endogenous competitive business cycles. *Econometrica*, 53, 994-1045.

- Jablanović, V. (2010). *Chaotic population growth*. Belgrade: Cigoja.
- Jablanović, V. (2011a). The chaotic saving growth model: G7. *Chinese Business Review*, 10(5), 317-327.
- Jablanović, V. (2011b). The chaotic monopoly price growth model. *Chinese Business Review*, 10(11), 985-990.
- Jablanović, V. (2012). *Budget deficit and chaotic economic growth models*. Aracne editrice. S.r.l..
- Li, T., & Yorke, J. (1975). Period three implies chaos. *American Mathematical Monthly*, 8, 985-992.
- Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of Atmospheric Sciences*, 20, 130-141.
- Lorenz, H. W. (1993). *Nonlinear dynamical economics and chaotic motion* (2nd ed.). Heidelberg: Springer-Verlag.
- May, R. M. (1976). Mathematical models with very complicated dynamics. *Nature*, 261, 459-467.
- Medio, A. (1993). *Chaotic dynamics: Theory and applications to economics*. Cambridge: Cambridge University Press.
- Medio, A. (1996). Chaotic dynamics. Theory and applications to economics. *De Economist*, 144(4), 695-698.
- Peitgen, H. O., Jürgens, H., & Saupe, D. (1992). *Chaos and fractals-new frontiers of science*. New York: Springer-Verlag.
- Rössler, O. E. (1976). An equation for continuous chaos. *Physics Letters A*, 57, 397-398.
- Tu, P. N. V. (1994). *Dynamical systems*. Verlag: Springer.